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Dynamics of two externally driven coupled quantum oscillators interacting with separate baths based on path integrals

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HIGHLIGHTS

- Time-dependent density matrix of two driven coupled oscillators interacting with separate reservoirs is derived based on path integration.
- All elements of the covariance matrix and mean values of observables are calculated.
- Temporal dynamics and stationarity of the oscillators from any given time up to quasi-equilibrium steady states are studied.

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ABSTRACT

The paper deals with the problem of dynamics of externally driven open quantum systems. Using the path integral methods we found an analytical expression for time-dependent density matrix of two externally driven coupled quantum oscillators interacting with different baths of oscillators. It is shown that at the zeroing of external forces the density matrix becomes identical to the previously obtained one for freely developing coupled oscillators. Mean values of observables and all elements of the covariance matrix composed by coordinates and momenta of two driven coupled oscillators are calculated. The time-dependent mean values, dispersions and covariances of coordinates of coupled oscillators at given external forces are numerically studied. It is shown that the larger the coupling constant the larger is the disturbances of the second oscillator due to external action on the first oscillator. Coupled dynamics of forced oscillators at relatively large coupling constant is demonstrated at different thermodynamic conditions.

1. Introduction

The physics of open systems covers various processes ranging from elementary-particle level to astrophysical scales. Also, such systems excite a great interest in technology and social sciences. An often-used model of open quantum systems consists of a quantum oscillator (set of oscillators) coupled to a heat reservoir (set of shared or separate reservoirs) of harmonic oscillators. This mechanical hamiltonian system allows investigating of dissipation, decoherence, correlation, continuous quantum measurement, quantum-to-classical transition and other important phenomena in nature. It is well-known that the model has been successful in describing the Brownian dynamics of selected particles coupled to a bath, see, for example, Refs. [1–19]. Study of the pure Hamiltonian composite system yields a reason of irreversibility in the dynamics of a selected quantum system interacting with surroundings after reduction with respect to reservoirs variables. It always

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deserves considerable attention because of transition from pure mechanical dynamics to thermodynamical laws due to inevitable effects of the environment on objects under study. Thus, the detailed description of relaxation processes of open systems to stationary or to quasi-stationary states, and to equilibrium or to quasi-equilibrium thermodynamic states is obviously very important.

For example, in the case of a harmonic oscillator with arbitrary damping and at arbitrary temperature an explicit expression for the time evolution of the density matrix when the system starts in a particular kind of pure state was derived and investigated in a seminal work [20] based on the path integral technique. It was shown that the spatial dispersion in the infinite time limit agrees with the fluctuation–dissipation theorem (FDT). To study the approach to equilibrium or to some transient stationary state of the system, a problem for coupled oscillators interacting with heat baths characterized by its own temperatures was considered in Refs. [21–26]. It was concluded that an arbitrary initial state of a harmonic oscillator state decays towards a stationary state. A study of a harmonic chain to whose ends independent heat baths are attached which kept at different temperatures was performed in Ref. [27]. It was found the chain approaches a stationary state regime. An analyze of the nonequilibrium steady states of a one-dimensional harmonic chain of atoms with alternating masses connected to heat reservoirs at unequal temperatures and a heat transfer across an arbitrary classical harmonic network connected to two heat baths at different temperatures were done in Refs. [28,29]. The evolution of quantum states of networks of quantum oscillators coupled with arbitrary external environments was analyzed in Ref. [30]. The emergence of thermodynamical laws in the long time regime and some constraints on the low frequency behavior of the environmental spectral densities were demonstrated.

Much attention has been focused on the bipartite continuous variable systems composed of two interacting oscillators.

Based on the non-Markovian master equations, the entanglement evolution of two harmonic oscillators under the influence of non-Markovian thermal environments was studied in Ref. [31]. It was demonstrated that the dynamics of the quantum entanglement is depended on the initial states, the coupling between oscillators and the coupling to shared baths or to separated baths. A study of the time-dependent entanglement and quantum discord between two oscillators coupled to a common environment was provided in papers [32–34]. Different stages of evolution including sudden disappearance and appearance of the entanglement were described providing a characterization of this process for different reservoirs including Ohmic, sub-Ohmic, and super-Ohmic models. Two coupled oscillators in a common environment at arbitrary temperature and the quantum decoherence of their states were investigated in Ref. [35]. It was shown that the problem can be mapped into that of a single harmonic oscillator in a general environment plus a free harmonic oscillator. Besides, simplest cases of the entanglement dynamics were considered analytically and an analytical criterion for the finite-time disentanglement was derived at the Markovian approximation. The time-dependent entanglement between two coupled different oscillators within a common bath and within two separate baths was studied in Ref. [36] based on a master equation. It was found that in the case of separate baths at not very low temperatures the initial two-mode squeezed state becomes separable accompanying with a series of features. For instance, if the two oscillators share a common bath, the observation of asymptotic entanglement at relevant temperatures becomes possible. The evolution of quantum correlations of entangled two-mode states in a single-reservoir and in a two-reservoir model was studied in Ref. [37]. It was shown that in the two-reservoir model the initial entanglement is completely lost, and both modes are finally uncorrelated, but in a common reservoir both modes interact indirectly via the same bath. In Ref. [38] a system of two coupled oscillators within separate reservoirs was investigated. It was shown that if the baths are at different temperatures, then the interaction between the particles must be strong enough in order to reach a steady state entanglement. No thermal entanglement between two coupled oscillators is found in the high-temperature regime and weak coupling limits in Ref. [39]. The existence of a nonequilibrium state for two coupled, parametrically driven, dissipative harmonic oscillators which has stationary entanglement at high temperatures was reported in Ref. [40]. Based on exact results for the non-Markovian dynamics of two parametrically coupled oscillators in contact to independent thermal baths, the out-of-equilibrium quantum limit derived in Ref. [40] is generalized to the non-Markovian regime in Ref. [41]. It is shown that non-Markovian dynamics allows for the survival of stationary entanglement at higher temperatures. A stationary regime of two coupled oscillators connecting with independent reservoirs of harmonic oscillators was studied in Ref. [42], and analytical formulas for the mean energy of interaction of the selected oscillators and their mean energies in this case were derived. Time-dependent behavior of variances and covariances of two coupled oscillators within separate baths in the weak-coupling limit was investigated in Ref. [43]. It was demonstrated that these characteristics of two weakly coupled oscillators in the infinite time limit agree with the FDT despite of initial variances. The case of arbitrary coupling of identical oscillators was considered in Ref. [44], and it was shown that the larger a difference in temperatures of thermal baths, the larger is a difference of the stationary values of variances of coupled identical oscillators as compared to values given by the FDT. The general case of two arbitrary coupled oscillators of arbitrary properties interacting with separate reservoirs is studied in Ref. [45]. As well as in previous cases the temporal dynamics of spatial variances and covariances of oscillators from any given time up to quasi-equilibrium steady states is studied based on path integration. It is shown for arbitrary oscillators that the spatial variances and covariances achieve stationary values in the long-time limit. It is demonstrated that the larger the difference in masses and eigenfrequencies of coupled oscillators, the smaller are the deviations of stationary characteristics from those given by the FDT at fixed coupling strength and fixed difference in temperatures between thermal baths.

The main goal of this paper is to derive an analytical expression for a temporary dependent density matrix of two selected oscillators subjected by two independent external forces at any arbitrary times. The reduced density matrices allow calculating whole set of elements of a covariance matrix. Temporal behavior of some mean values of observables is provided.