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Exponential model for option prices: Application to the Brazilian market

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HIGHLIGHTS

- Daily returns on the Ibovespa index follow an exponential distribution.
- Comparison is made between the Black-Scholes and the exponential models for option pricing.
- Near maturity, option prices are better described by the exponential model.
- Possible implications for investment strategies and risk management are briefly discussed.

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ABSTRACT

In this paper we report an empirical analysis of the Ibovespa index of the São Paulo Stock Exchange and its respective option contracts. We compare the empirical data on the Ibovespa options with two option pricing models, namely the standard Black–Scholes model and an empirical model that assumes that the returns are exponentially distributed. It is found that at times near the option expiration date the exponential model performs better than the Black–Scholes model, in the sense that it fits the empirical data better than does the latter model.

1. Introduction

Options are financial instruments that allow their holders the right of buying or selling the underlying asset for a given fixed price, also known as the strike price, on a pre-defined date called the expiration or maturity date. The central issue of modeling option prices is to determine the fair price, also called premium, to pay for a given option before the maturity date, taking into account the statistical distribution of the underlying asset.

The formulation of the Black, Scholes and Merton model in the 1970's established a landmark for option pricing [1]. This model assumes that asset prices in financial markets can be described by a geometric Brownian motion. This hypothesis is the backbone of the so-called *Efficient Market Hypothesis* (EMH), which asserts that the returns of a given stock follow an uncorrelated Gaussian process (white noise).

However, studies on high frequency financial data (e.g., on time scales of the order of a few minutes or less) have shown that price fluctuations behave as non-Gaussian processes [2–4], with the empirical probability distribution function (EDF) of

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returns exhibiting power law tails. On the other hand, on mesoscopic time scales (typically from hours to days) the central part of the EDFs are often better described by an exponential distribution [5–10]. At longer time lags, the EDFs tend to a Gaussian distribution as required by the central limit theorem. Given this scenario, it is natural to ask how option prices should be modeled when the distribution of returns of the underlying asset displays such a variability.

Several non-Gaussian option pricing models have been considered in the literature [11,12]. These include models based on Lévy processes [13,14] and on the so-called q-Gaussian distribution [15], in which cases the distribution of returns have power-law tails; stochastic volatility models [16,17], including the Hull–White and Heston models [18,19]; option pricing with the Edgeworth expansion [20,21]; etc. Of particular relevance to us here is the empirical model for option pricing introduced by McCauley and Gunaratne [22] which assumes an exponential distribution of returns. As mentioned above, financial markets often exhibit exponential distributions on time scales that are comparable to the lifetime of an option, and so the exponential model for option pricing is a natural candidate to describe options in such markets. In contradistinction, power-law distributions tend to be observed in high-frequency data (e.g., intraday quotes) and are thus expected to be less relevant for option pricing in such cases.

In the present paper we perform an empirical analysis of the prices of options on the Ibovespa index of the São Paulo Stock Exchange. First we show that, as in other financial markets [6–9], the central part of the distribution of the Ibovespa returns follows an exponential distribution at mesoscopic times scales. We then proceed to analyze the Ibovespa option market in light of two relevant option pricing models: (i) the standard Black–Scholes model [1] and (ii) the exponential model for option pricing [22] mentioned above. Both models yield an analytical solution for the price of an European call option, which can be easily compared with the quoted market prices. We find that the exponential model gives a better fit to the empirical data for times closer to the option expiration date, whereas for longer periods before expiration the Black–Scholes model offers a better description of the market prices. Our findings thus indicate that the market seems to implicitly take into account the fact that the returns of the Ibovespa (at mesoscopic time scales) follow an exponential distribution. Implications of this finding for possible investment strategies are briefly discussed.

2. Exponential model for option pricing

In this section we collect the main results concerning the exponential distribution for financial returns and its applications to option pricing.

2.1. Distribution of returns

Let us define the logarithmic returns at time lag τ by $x(t) = \ln[S(t+\tau)/S(t)]$, where $S(t)$ is the price of the relevant financial asset at time t and τ is the time lag. The exponential distribution, $f(x, t)$, of the log-returns x is defined by [22]

$$f(x,t) = \begin{cases} Ae^{\gamma(x-\delta)} & \text{if } x \le \delta \\ Be^{-\nu(x-\delta)} & \text{if } x > \delta, \end{cases} \tag{1}$$

where δ, γ and ν are parameters that characterize the distribution. From the normalization condition, $A/\gamma + B/\nu = 1$, and by imposing $\langle x\rangle = \delta$, one finds that $A/\gamma^2 = B/\nu^2$. One can also show that the variance of the exponential distribution is given by $2(\gamma\nu)^{-1}$; see Ref. [22] for details.

Let us now recall that the folded cumulative distribution, $G(x)$, associated with a probability density function $f(x)$ is defined by

$$G(x) = \begin{cases} F(x), & \text{if } F(x) \le \dfrac{1}{2} \\ 1 - F(x), & \text{otherwise} \end{cases} \tag{2}$$

where $F(x)$ is the cumulative distribution: $F(x) = \int_{-\infty}^{x} f(x)dx$. Note that $G(x)$ for the exponential distribution given in (1) is also a bilateral exponential function. This means, in particular, that in a semi-log plot the function $G(x)$ for the exponential distribution has a tent-like shape, in contradistinction to a Gaussian distribution whose folded cumulative distribution has a downward concavity.

In Fig. 1(a) we plot in a semi-log scale the empirical folded cumulative distribution of the Ibovespa returns for $\tau = 1, 5$ and 20 days. These distributions were generated from the historical time series of the daily closing prices of the Ibovespa, from its inception in January 1968 up to February 2004, totaling 8889 data points. One sees from this figure that the empirical distributions (solid lines) deviate from a Gaussian (dashed line) and follow instead an exponential law, particularly in the central region of the EDF which displays the tent-like shape typical of an exponential distribution.

At shorter time scales, the distribution of returns of the Ibovespa becomes heavy-tailed as seen in Fig. 1(b), where we plot the folded cumulative distribution of intraday returns for $\tau = 15$, 60, and 180 min. The distributions in this figure were constructed from a time series of 19995 intraday quotes at every 15 min covering the years from 1998 to 2001. Note, in particular, that for $\tau = 15$ min the distribution has an upward concavity typical of power law distributions. However, as τ increases the empirical distribution evolves towards an exponential distribution, as seen in the case for $\tau = 180$ min