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Q1 Evolutionary systemic risk: Fisher information flow metric in financial network dynamics

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HIGHLIGHTS

- Apply Fisher information metric to construct financial networks.
- Introduce the evolution index (EI) as a quantitative measure of network evolution.
- Empirically, systemic risk of stock market is high when EI runs below its long-term average.
- The change of EI Granger causes the change of stock price.

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ABSTRACT

Recently the topic of financial network dynamics has gained renewed interest from researchers in the field of empirical systemic risk measurements. We refer to this type of network analysis as information flow networks analysis (IFNA). This paper proposes a new method that applies Fisher information metric to the evolutionary dynamics of financial networks using IFNA. Our paper is the first to apply the Fisher information metric to a set of financial time series. We introduce Evolution Index (EI) as a measure of systemic risk in financial networks. It is shown, for concrete networks with actual data of several stock markets, that the EI can be implemented as a measure of fitness of the stock market and as a leading indicator of systemic risk.

1. Introduction

Financial network is a major branch of econophysics using statistical analysis of stock networks and their structural dynamics. Financial network dynamics refers to how a financial network evolves as a function of time. Systemic risk of a given network N denoted by SR(N), can be defined in many ways. We take the simple definition that the systemic risk of a network SR(N) is the probability of an event or a sequence of events rendering the network inoperable. The system itself must be defined by the experiment with available empirical data. For example, in financial networks "inoperable" means that cash flows and liquidity in the network drop below a certain threshold. We seek methods that are versatile and scalable represented by an evolutionary index as aggregate proxy for SR(N).

Our approach in this paper relies on the premise that a good assessment of cash flows in a financial network must be based on information flows in the same network. We refer to this type of network analysis as information flow networks analysis (IFNA). This paper proposes a new method to show the evolutionary dynamics of financial networks using IFNA. There are several information metrics that researchers can use to measure distances between signals, time series and general variables described by distributions. In this paper we demonstrate that the Fisher information metric provides a metric on financial

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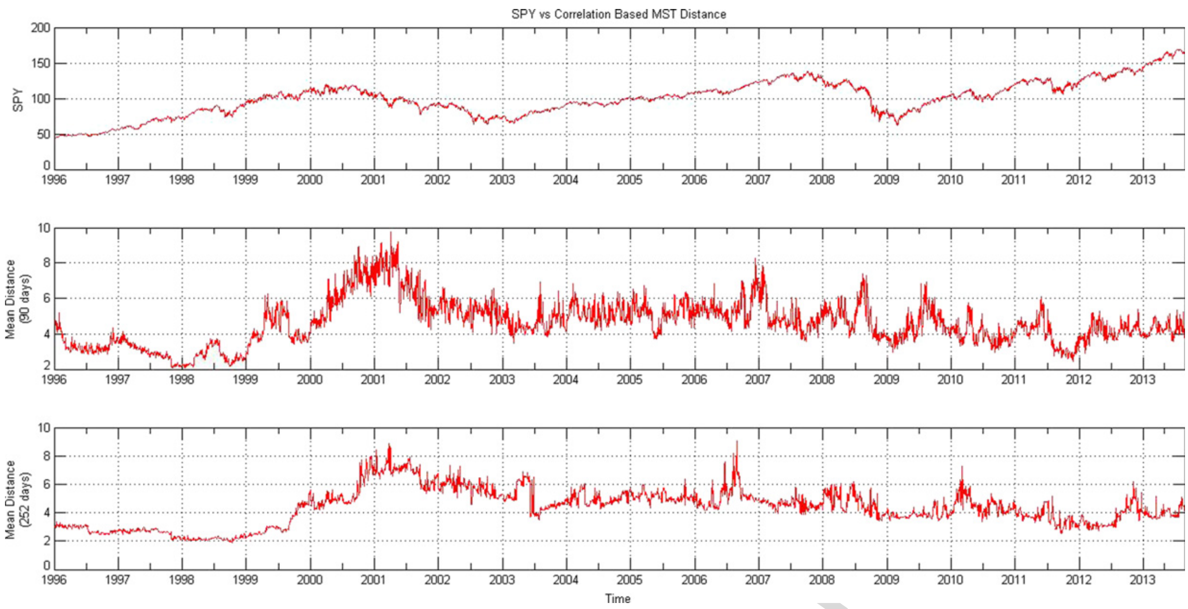

Fig. 1. SPDR S&P 500 Index ETF (SPY) and its mean distance of MST $\overline{d_{ij}} = \sqrt{2\left(1-\rho_{ij}\right)}$.

time series and refer to this method as IFNA(F). Moreover, it is discovered that once the financial network relative distances are calculated according to IFNA(F), we can construct minimum spanning trees, which we denote MST(F). Therefore, IFNA(F) calculates the relative distances in the network based on Fisher information metric and MST(F) maps this network onto its corresponding minimum spanning tree. In the empirical analysis, we suggest that a network that maximizes its Fisher information metric tends towards similarity. Based on that observation, we introduce an Evolution Index (EI) based on the MST(F). It is shown, for concrete network with actual stock market data of US, North America, Europe and Asia, that the EI can be viewed as a measure of fitness of the stock market.

The minimum spanning tree (MST) problem was firstly introduced in 1926 [1] and its efficient solutions were found by Refs. [2,3]. In financial area, statistical analysis of stock interaction networks and their structural dynamics using the correlation distance of $d_{ij} = \sqrt{2\left(1-\rho_{ij}\right)}$ where ρ_{ij} is the Pearson correlation coefficient between the return time series of stock i and stock j was introduced in Ref. [4]. It is found that the structure of the stock market MST evolves slowly and locally and that such non-randomness is a robust property of stock market topology [5]. Transfer entropy was also used to construct an MST and shows that it can measure the information flow among global stock markets [6]. There are many empirical studies that employ the correlation based MST in stock market [7–14]. In forex and future markets, we refer to Refs. [15–17].

The novelty of this paper is twofold. First we employ information-theoretic approach to analyze evolutionary systemic risk dynamics in a financial network. Second we introduce a new information theoretic metric for the MST instead of the correlation based metric $d_{ij} = \sqrt{2\left(1-\rho_{ij}\right)}$. The disadvantage of the Pearson correlation based MST is that it can only capture the strength of a linear relation and, as a result, it is insensitive to the dynamics of the stock market since most of the financial time series are non-linear. It is shown that the average distance of MST can hardly detect market growth or drawdown when we apply the correlation based MST with a rolling window of $T = 90$ and $T = 252$ days from 1996/01/02 to 2013/08/30 (Fig. 1). We show in the next paragraph that the Fisher information metric provides superior interpretation of network dynamics capturing the non-linear properties between probability distributions.

2. Fisher information metric

Information geometry is the application of the field of differential geometry to probability theory. The Fisher information metric is a special case of Riemann metrics on a smooth manifold representing probability distributions. The Fisher metric induced by the inner product can be written in terms of Poincaré–Beltrami half-plane distance, which is a classic example of non-Euclidean hyperbolic geometry [18].

In this context the term *systemic evolution* refers to the statistical aggregation of component indicators into a collective vector of measurements that informs on the system. It should not be surprising to think of the evolution of the system and think about a model that captures systemic evolution *quantitatively* since there should be valuable information pertaining to systemic risk in observing an evolving system. Heiberger gave several quantitative measurements of financial systemic evolution as May–Wigner stability, modularity, clustering coefficient and assortativity [13]. However, their network is built

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