Physica A 445 (2016) 75–84

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Gaussian fidelity distorted by external fields

Jonas F.G. Santos *, Alex E. Bernardini

Departamento de Física, Universidade Federal de São Carlos, PO Box 676, 13565-905, São Carlos, SP, Brazil

HIGHLIGHTS

- Quantum information is quantified through the quantum fidelity and Shannon entropy.
- The Gaussian fidelity for three physical systems in the presence of a magnetic field is studied.
- A map between the external magnetic field and the non-commutative parameters is established.

ARTICLE INFO

Article history:
Received 25 August 2015
Available online 10 November 2015

Keywords:
Quantum fidelity
Wigner function
Quantum information
Shannon entropy

ABSTRACT

Gaussian state decoherence aspects due to interacting magnetic-like and gravitational fields are quantified through the quantum fidelity and Shannon entropy in the scope of the phase-space representation of elementary quantum systems. For Gaussian Wigner functions describing harmonic oscillator states, an interacting external field destroys the quantum fidelity and introduces a quantum beating behavior. Likewise, it introduces harmonic profiles for free particle systems. Some aspects of quantum decoherence for the quantum harmonic oscillator and for the free particle limit are also quantified through the Shannon entropy. For the gravitational quantum well, the effect of a magnetic-like field on the quantum fidelity is suppressed by the linear term of the gravitational potential. To conclude, one identifies a fine formal connection of the quantum decoherence aspects discussed here with the noncommutative quantum mechanics.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

Gaussian quantum correlations are in the core of the quantum information issues involving continuous variable systems. In addition, Gaussian states are also the elementary blocks in building vacuum states, thermal states and coherent states [1], as they play a fundamental role in quantum optics, in low dimensional physics, or even as an effective tool for describing atomic ensembles [2]. Besides providing the necessary theoretical tools for the understanding and the manipulation of quantum correlated systems, the representation of Gaussian states through the Wigner formalism works as bridge to the classical dynamics. From a phenomenological point of view, Gaussian Wigner functions can be parametrically manipulated as to describe a set of measured data [3,4] circumstantially correlated to the issues of quantum–classical transitions of a physical system [5], for instance, as an indicator of quantum chaos [6].

The inclusion of external fields into the Hamiltonian that drives the behavior of Gaussian states may bring up typical decoherence and dissipation with recognized phenomenological appeal. External fields acting on specific quantum systems are frequently implemented through of quantum simulations, where a kind of controllable quantum system is used to study another less accessible one [7]. As a typical example, the effect of a constant magnetic field on atoms has revealed the split in the energy spectrum, in the well-known phenomenon of Zeeman effect [8].

* Corresponding author.
E-mail addresses: jonas@df.ufscar.br (J.F.G. Santos), alexeb@ufscar.br (A.E. Bernardini).

http://dx.doi.org/10.1016/j.physa.2015.10.033
0378-4371/© 2015 Elsevier B.V. All rights reserved.

Given that the Wigner function in the phase-space quantum mechanics is connected with the information which can be obtained from a quantum system, the quantum fidelity, computed through a Gaussian envelop, can encompass the decoherence aspects of the dynamical evolution of such elementary quantum systems [9,10]. In this letter, the influence of magnetic external fields on the quantum fidelity of two dimensional harmonic oscillators, their corresponding free particle limit, and an extension as to include the gravitational quantum well (GQW) dynamics, are therefore quantified through the Gaussian Wigner formalism.

Our manuscript is organized as follows. In Section 2 the Wigner–Weyl formalism of the quantum mechanics, as well as the way to obtain the quantum fidelity and the Shannon entropy are introduced. The Gaussian Wigner state to be used throughout the subsequent analysis of some particular quantum systems is presented. In Section 3, one reports about the Wigner phase-space formalism for the harmonic oscillator in the presence of an external interacting magnetic field. The classical limit for the free particle system is obtained by setting null the natural frequency of the oscillator, $\omega_0 = 0$. The Gaussian state fidelity and a qualitative analysis for the Shannon entropy are also evaluated. In Section 4, the quantum fidelity and the quantum decoherence aspects for the gravitational quantum well are discussed in the phase-space framework. A fine formal connection between external magnetic field interacting systems and the noncommutative quantum mechanics is noticed in Section 5. Our conclusions are drawn in Section 6.

2. The Weyl–Wigner formalism of the quantum mechanics, quantum fidelity, and Shannon entropy

By identifying the density matrix of a quantum system with $\hat{\rho} = |\Psi\rangle\langle\Psi|$, one can define a Wigner function through the Weyl transform as [11,12],

$$W(r, p) = h^{-1}\rho^W = \int \mathrm{d}s\, \exp[i\, p\, s/\hbar]\, \Psi(r - s/2)\, \Psi^*(r + s/2), \tag{1}$$

which can be naturally generalized to a statistical mixture, such that the expectation value of an observable $\hat{O}$ can be computed through

$$\langle O\rangle = \iint \mathrm{d}r\, \mathrm{d}p\, W(r, p)\, O^W(r, p). \tag{2}$$

The probability distributions for r and p are equivalently given by

$$\int \mathrm{d}r\, W(r, p) = \Phi^*(p)\, \Phi(p) \quad \text{and} \quad \int \mathrm{d}p\, W(r, p) = \Psi(r)^*\Psi(r), \tag{3}$$

such that the Wigner function can also be computed from $\Phi(p)$ through

$$W(r, p) = \int \mathrm{d}s\, \exp[-i\, r\, s/\hbar]\, \Phi(p - s/2)\Phi^*(p + s/2). \tag{4}$$

Additional properties related to the density matrix theory can be obtained from the above prescription [13,14]. For instance, the Weyl transform of an operator has intrinsic properties that allow one to write the quantum fidelity, F, in terms of Wigner functions. The quantum fidelity, F, is a commonly used measure to compare an input state and an output state through a given quantum channel [1,15,16], as it works as a kind of decoherence quantifier. If F goes to unity, it means that the output state is very similar to the input state. Likewise, if F goes to zero, the output is completely different from the input state. Effectively, the fidelity measures the projection, varying from zero to unity, of a time-evolving state onto a departure state.

By using the Weyl transform of an operator and the property of the trace of the product of two operators, one has

$$Tr[\hat{A}\hat{B}] = \iint A^W(r, p)\, B^W(r, p)\, \mathrm{d}r\mathrm{d}p, \tag{5}$$

which can be used into the definition of the quantum fidelity [1],

$$F(\hat{\rho}_1, \hat{\rho}_2) = \left[Tr(\sqrt{\sqrt{\hat{\rho}_1}\hat{\rho}_2\sqrt{\hat{\rho}_1}})\right]^2, \tag{6}$$

where $\hat{\rho}_1$ and $\hat{\rho}_2$ are two states of the quantum system. Noticing that

$$Tr[(\hat{\rho}_1\hat{\rho}_2)^{1/2}] = \iint (\rho_1^W \rho_2^W)^{1/2}\, \mathrm{d}r\mathrm{d}p, \tag{7}$$

and using the Wigner function from Eq. (1), one obtains

$$Tr[(\hat{\rho}_1\hat{\rho}_2)^{1/2}] = \iint (W_1 W_2)^{1/2}\, \mathrm{d}r\mathrm{d}p, \tag{8}$$

Download English Version:

https://daneshyari.com/en/article/7378552

Download Persian Version:

https://daneshyari.com/article/7378552

Daneshyari.com