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Concentration of diffusional particles in viscous boundary sublayer of turbulent flow

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HIGHLIGHTS

- We present a statistical model for particle dispersion in inhomogeneous random flow.
- The model describes the particle statistics in its dependence of Stokes number.
- The power-law density profile for particles in viscous boundary sublayer is found.
- Sufficiently inertial particles are localized in the viscous boundary sublayer.

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ABSTRACT

The motion of inertial particles suspended in the viscous boundary sublayer of a wall-bounded turbulent flow is considered. We show that there is the diffusional region near the wall occupied by the particle moving in the local-equilibrium with the turbulent fluctuations which are modelled by Ornstein–Uhlenbeck process. The stationary spatial distribution of inertial particles is found for arbitrary Stokes number (dimensionless measure of the inertia). The weakly inertial particles placed in the diffusional region go away from the wall and escape the viscous sublayer. However, the direction of particle migration reverses when the Stokes number is getting larger than the critical value that we determine. The standard turbophoresis becomes more and more pronounced with the increase of Stokes number so that sufficiently inertial particles turn out to be trapped in the viscous boundary sublayer.

1. Introduction

The study of turbulent suspensions of inertial particles is of great importance for understanding of various industrial and environmental processes [1,2]. One of the most interesting phenomena associated with turbulent transport is the formation of nonuniform spatial distribution of particles in inhomogeneous turbulence. There is the widely known turbophoretic effect: inertial particles migrate in the direction of decreasing turbulence level and accumulate in minima of turbulence intensity [3–9]. On the other hand, recent theoretical study [10] indicates that turbophoresis reverses sign for the sufficiently heavy particles whose time of equilibration is longer than the time to reach the minimum of turbulence. Apparently, this result will motivate the further investigations on the subject.

A natural example of a spatially nonuniform random flow is the near-wall region of developed high-Reynolds hydrodynamic turbulence [11]. The intensity of fluid velocity fluctuations tends to zero at the wall due to the non-slip boundary

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conditions. As a result, the presence of the boundary produces strong spacial inhomogeneity and anisotropy of the flow. Inertial particle statistics in wall-bounded turbulent fluid is a subject of numerous investigations due to its importance in different physical contexts: particle deposition process, particle accumulation effect, heat and mass transfer [1,3,12–14].

In the present theoretical study we examine the particle dispersion at the viscous scale of near-wall turbulence, i.e. inside the viscous (laminar) boundary sublayer. The previous theoretical works on the subject are mainly devoted to the case of passive tracers [15,16] or to the limit of very inertial particles [17]. Here we propose a method of statistical analysis which allows us to describe the particle statistics in its dependence on the particle inertia. We demonstrate that there is the diffusional region near the wall where particles move in local-equilibrium with the fluctuations of fluid velocity. Using Ornstein–Uhlenbeck process as a simple model of random flow we derive the closed equation on the particle real space concentration which is valid at arbitrary particle inertia. The equilibrium concentration profile of the diffusional particles has a power-law form with the exponent determined by the particle Stokes number. Our analysis reveals the localization–delocalization transition as the effect of inertia is increased. Being injected in the diffusional region of viscous sublayer inertial particles whose Stokes number is larger than some critical value are accumulated near the wall. On the contrary, weakly inertial particles tend to escape the near-wall region. Thus, the theory predicts that particles with different Stokes numbers disperse in the viscous boundary sublayer of turbulent flow.

The paper is organized as follows. In the next section we develop the theoretical framework that provides description of the particle real space concentration in a spatially nonuniform random flow. In Section 3 this approach is applied to the inertial particles placed in the viscous boundary sublayer of wall-bounded turbulence. Finally, in Section 4 we summarize the results and present some general remarks.

2. Particle real space concentration

2.1. General relations

Consider the motion of inertial particles embedded in a random flow $\vec{u}(\vec{r}, t)$ of an incompressible fluid. Every particle is assumed to be much heavier than the fluid and so small that the flow around it is viscous. We neglect the inertia of the displaced fluid, interaction between particles and their influence on the flow. Then, the particle velocity $\vec{v}$ changes according to

$$\frac{d\vec{v}}{dt} = \frac{1}{\tau}(\vec{u} - \vec{v}), \tag{1}$$

where τ is the particle response time [18].

The fluid velocity $\vec{u}(\vec{r}, t)$ is treated as a random function of time which has to be characterized statistically. The flow is assumed to be homogeneous in time, whereas there is no homogeneity in space: typical amplitude of velocity fluctuations depends on space coordinates. We mainly specialize on the case when the fluid flow is shortly correlated in time and the particles experience strong viscous damping so that the characteristic timescales of both fluid's and particle's velocities dynamics (i.e. τ_c and τ) are much smaller compared to the temporal scales associated to the evolution of the particle's positions. Therefore, there are two small parameters in our model: the correlation time of the random velocity field τ_c, and the particle response time τ. The ratio of these two timescales $\mathrm{St} = \tau/\tau_c$, which is known as the Stokes number, is a dimensionless measure of particle inertia. Below we demonstrate that being taken successively the limits $\tau_c \to 0$ and $\tau \to 0$ do not commute in the case of spatially inhomogeneous flow. Different orders of these limits correspond to opposite physical situations ($\mathrm{St} \to 0$ and $\mathrm{St} \to \infty$) and give different transport equations on particle concentration. However, we do not restrict particle Stokes number considering in detail these limiting cases as well as a general case of relation between the particle response time and the fluid velocity correlation time.

2.2. Small Stokes number

First, let us assume the particle response time to be much smaller than the fluid velocity correlation time, $\mathrm{St} \ll 1$. In that case the inertia can be neglected in the process of particle transport by random velocity field. Neglecting the term $\dot{\vec{v}}(t)$ in (1) one finds the first-order stochastic equation

$$\dot{\vec{r}}(t) = \vec{u}(\vec{r}(t), t), \tag{2}$$

that describes the particle dynamics on timescales $t \gg \tau$. The probability density of a particle distribution in real space is $n(\vec{r}, t) = \langle \delta(\vec{r} - \vec{r}(t)) \rangle$ where the trajectory $\vec{r}(t)$ is a particular solution of Eq. (2) and the averaging is over the statistics of random flow. We focus on the case when the characteristic timescale of evolution of the positional distribution is very large compared to the fluid velocity correlation time τ_c. Then, treating the incompressible velocity field $\vec{u}(\vec{r}, t)$ in (2) as a white noise one can perform standard derivation of the Fokker–Planck equation [19]. If there is no net fluid flow in any direction, $\langle u_i(\vec{r}, t) \rangle = 0$, the result is

$$\partial_t n = \partial_{r_i}[D_{ij}(\vec{r}, \vec{r})\partial_{r_j} n], \tag{3}$$