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Multifractal detrended fluctuation analysis based on fractal fitting: The long-range correlation detection method for highway volume data

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HIGHLIGHTS

- Investigating the traffic time series for volume data observed on the Guangshen highway.
- Proposing multifractal detrended fluctuation analysis based on fractal fitting (MFDFA-FF) to detect long-range correlations of time series.
- Fractal fitting can get a better detrend effect than polynomial fitting.
- MFDFA-FF is a more appropriate method for time series to detect the long-range correlation and multifractality.

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ABSTRACT

In this paper, we investigate the traffic time series for volume data observed on the Guangshen highway. We introduce a multifractal detrended fluctuation analysis based on fractal fitting (MFDFA-FF), which is one of the most effective methods to detect long-range correlations of time series. Through effective detecting of long-range correlations, highway volume can be predicted more accurately. In order to get a better detrend effect, we use fractal fitting to replace polynomial fitting in detrend process, the result shows that fractal fitting can get a better detrend effect than polynomial fitting and the MFDFA-FF method can achieve a more accurate research result. Then we introduce the Legendre spectrum to detect the multifractal property characterized by the long-range correlation and multifractality of Guangshen highway volume data.

1. Introduction

Fractal was proposed by Mandelbrot in 1975 [1]. So far, the mathematical definition of fractal is not strict, which only has a descriptive definition. Fractal is a rough or fragmented geometrical shape that can be subdivided into parts, each of which is a reduced-size copy of the whole, a fractal system is usually described by a scale invariant parameter called fractal dimension [2]. Although there is not a unified, strict definition about "fractal", it has received extensive attention of many disciplines as a new analysis method. Relevant empirical studies have made many valuable results. Using fractal theory to study the nature of the complex system has become one of the most widely discussed fields recently.

Many fractals arising in nature have a far more complex scaling relation and require a set of parameters to describe characteristics of complex systems that are known as multifractals. It is well-known that the formation, occurrence and

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development process of many natural phenomena are not strictly random, but have correlation and predictability. Due to the difficulty of analysis in achieving long-range correlations of nonstationary series, Peng et al. [3] proposed an approach to analyze the DNA sequences which are known as detrended fluctuation analysis (DFA). Stanley et al. [4–6] researched deeply on detrended fluctuation analysis (DFA). DFA method has been applied to many fields, such as DNA structure [7], records of the heart [8] and meteorological data [9]. Although the DFA method is widely used to achieve monofractal scaling properties, it cannot properly describe multifractal of time series. Based on the DFA method, Kantelhardt et al. [10] introduced the multifractal detrended fluctuation analysis (MFDFA) for the multifractal characterization of nonstationary time series. MFDFA can be used to establish a scientific and reasonable prediction, forecasting model. The fractals have been applied to studying traffic time series in the last several years [11–13]. As a powerful technique, MFDFA has so far been applied to the time series analysis in traffic fields [14]. For traffic time series, to better estimate the long-range correlations and its multifractal properties, MFDFA based on fractal fitting called MFDFA-FF is proposed.

Another fundamental concern is to study the possible fractal structure of long-range correlation, the Legendre spectrum was extensively studied. Dai et al. [15] used Legendre spectrum and Hölder exponent to analyze multifractal and singularity of Guangshen highway volume data. In this paper, we use MFDFA-FF method to achieve the Hurst exponent $h(q)$ which related to the classical multifractal scaling exponent $\tau(q)$. MFDFA-FF is a more appropriate method for time series to detect the long-range correlation and multifractality. Note that unlike the previous paper, based on MFDFA-FF method, we can achieve the value of Hurst exponent more accurately to detect the long-range correlation and generate the Legendre spectrum more precisely to analyze the multifractal property.

The organization of this paper is as follows. Firstly, we introduce the fractal interpolation and fractal fitting in Section 2 and multifractal detrended fluctuation analysis based on fractal fitting (MFDFA-FF) in Section 3. Secondly, the detrended ability analysis with MFDFA-FF shows that MFDFA-FF can achieve a better detrend effect than MFDFA in Section 4. Thirdly, we introduce the multifractal spectrum in Section 5. Then we analyze the long-range correlation and multifractal of Guangshen highway volume data in Section 6. Finally, we make a conclusion.

2. The fractal interpolation and fractal fitting

Fractal interpolation has been developed as an alternative interpolation technique suitable for data with inherent fractal structure. Given interpolation points $\{(x_i, y_i)\}_{i=0}^{N}$, fractal interpolation is based on the theory of iterated function systems (IFS) that is convenient for fitting physical or experimental data [16,17]. In contrast to fractal interpolation, traditional interpolation is built on elementary functions such as polynomials. Fractal interpolation function has been used in fitting the unsmoothed curve which shows the superiority to other functions. Fractal interpolation function is becoming a research field for approximation theory of functions.

A fractal interpolatory scheme can be introduced in the following way [16].

Let $I = [a, b]$. $a = x_0 < x_1 < \cdots < x_N = b$ is a partition for I and $N \geq 2$. $y_0, y_1, \ldots, y_N$ are arbitrary real numbers. Let $K = I \times R$ and $I_i = [x_{i-1}, x_i]$, $i = 1, 2, \ldots, N$. L_i is a compression homeomorphism for $I \to I_i$

$$L_i(x_0) = x_{i-1}, \qquad L_i(x_N) = x_i, \tag{1}$$

and for any $u_1, u_2 \in I$

$$|L_i(u_1) - L_i(u_2)| \leq l_i \cdot |u_1 - u_2|,$$

where $0 < l_i < 1$.

F_i is a continuous function for $K \to R$

$$F_i(x_0, y_0) = y_{i-1}, \qquad F_i(x_N, y_N) = y_i, \tag{2}$$

and for any $u \in I$, v_1, $v_2 \in R$

$$|F_i(u, v_1) - F_i(u, v_2)| \leq q_i \cdot |v_1 - v_2|,$$

where $0 \leq q_i < 1$.

w_i is a mapping $K \to K$

$$w_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} L_i(x) \\ F_i(x, y) \end{pmatrix}, \quad i = 1, 2, \ldots, N. \tag{3}$$

So far, IFS $\{K; w_i, i = 1, 2, \ldots, N\}$ has been constructed. Specially, when $L_i(x)$ and $F_i(x, y)$ are linear functions, Eq. (3) is as follows,

$$w_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}, \quad i = 1, 2, \ldots, N, \tag{4}$$

satisfying

$$L_i(x_0) = x_{i-1}, \qquad L_i(x_N) = x_i, \qquad F_i(x_0, y_0) = y_{i-1}, \qquad F_i(x_N, y_N) = y_i, \tag{5}$$