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Pattern dynamics in a vegetation model with time delay ☆

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HIGHLIGHTS

- A vegetation model with both cross diffusion and time delay is considered.
- There exists a threshold of time delay in predator–prey interactions.
- The spatial patterns via numerical simulations are illustrated.

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ABSTRACT

A vegetation model with both cross diffusion and time delay is considered. Based upon a stability analysis, we demonstrate that the delay affects the stability and spatial patterns under some conditions. In addition, through numerical simulations, we obtain different spatial patterns, including spot patterns and stripe-like patterns. These results may help us better understand the dynamics of a vegetation model with time delay.

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1. Introduction

Vegetation plays a critical role in life. On the one hand, vegetation absorbs carbon dioxide and releases oxygen, which maintains the balance of the ecosystem. On the other hand, vegetation evaporates soil moisture into the atmosphere, which promotes the global water circulation cycle. Klausmeier proposed a very simple partial differential equation model with equations for surface water $R(X, Y, T)$ and for vegetation $U(X, Y, T)$ [1]:

$$\begin{aligned} \frac{\mathrm{d}R}{\mathrm{d}T} &= d - eR - fU^2R, \\ \frac{\mathrm{d}U}{\mathrm{d}T} &= aU^2R - bU, \end{aligned} \tag{1.1}$$

where a is the plant growth rate, b is the plant loss rate, d is the amount of rainfall, e is the water evaporation rate, and f is the uptake rate of the plant.

According to Sherratt [2,3], through a non-dimensional transformation

$$\gamma = R\frac{a}{\sqrt{ef}}, \qquad u = U\sqrt{\frac{f}{e}}, \qquad t = Te,$$

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we arrive at the following equations:

$$\frac{d\gamma}{dt} = S - \gamma - u^2\gamma, \quad \frac{du}{dt} = u^2\gamma - Bu, \tag{1.2}$$

where

$$S = ade^{3/2}\sqrt{f}, \qquad B = b/e.$$

In most practically relevant cases, the state of the ecosystem can exhibit temporal long-range correlations and should first be affected by its immediate past, with additional correction arising from time delays. In practice, time delays always exist and play a significant role in the dynamics; for example, time delays can be used to introduce oscillations [4–8].

When combined with the time delay factor, the model is given by

$$\frac{\partial\gamma}{\partial t} = S - \gamma - [u(t-\tau)]^2\gamma, \quad \frac{\partial u}{\partial t} = [u(t-\tau)]^2\gamma - Bu(t-\tau), \tag{1.3}$$

where $\tau > 0$ is a constant delay due to gestation.

Spatial patterns are ubiquitous in nature. These patterns modify the temporal dynamics and stability properties of population densities over a range of spatial scales. Their effects must be incorporated in temporal ecological models that do not represent space explicitly. When combined with spatial factor and diffusion terms, the original spatially extended model is written as the following system

$$\frac{\partial\gamma}{\partial t} = S - \gamma - [u(t-\tau)]^2\gamma + d_1\nabla^2\gamma \quad \frac{\partial u}{\partial t} = [u(t-\tau)]^2\gamma - Bu(t-\tau) + d_2\nabla^2 u, \tag{1.4}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}$ or $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the typical Laplacian operator in one- or two-dimensional space. d_{12} and d_{21} are the cross-diffusion coefficients of surface water and vegetation, respectively.

However, to the best of our knowledge, little work has been conducted on the dynamical behavior of both time delay and diffusion in a vegetation model. Therefore, in the present study, our objective is to investigate a vegetation model with both cross diffusion and time delay. More specifically, the primary objective of the present study is to investigate the spatial patterns.

2. Analysis

In this section, we consider the dynamics of model (1.4) with discrete time delay. We determined that model (1.4) and model (1.2) possessed the same positive equilibrium point $E^*(\gamma^*, u^*)$, where

$$\gamma^* = \frac{S - \sqrt{S^2 - 4B^2}}{2}, \quad u^* = \frac{2B}{S - \sqrt{S^2 - 4B^2}}.$$

It is easy to obtain that the condition for ensuring that γ^* and u^* are positive is that $S > 2B$.

We always assume that (γ^*, u^*) is linearly stable, thus, the eigenvalues of the Jacobian

$$\mathbf{A} = \begin{pmatrix} \frac{\partial f}{\partial \gamma} & \frac{\partial f}{\partial u} \\ \frac{\partial g}{\partial \gamma} & \frac{\partial g}{\partial u} \end{pmatrix}_{(\gamma^*, u^*)} \triangleq \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} -\frac{2B}{S - \sqrt{S^2 - 4B^2}} & -2B \\ \frac{4B^2}{(S - \sqrt{S^2 - 4B^2})^2} & B \end{pmatrix}$$

at (γ^*, u^*) must have negative real parts, which is equivalent to

$$tr(A) = a_{11} + a_{22} < 0, \tag{2.1}$$

and

$$det(A) = a_{11}a_{22} - a_{12}a_{21} > 0. \tag{2.2}$$

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