Physica A 444 (2016) 1012–1026

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Stable continuous-time autoregressive process driven by stable subordinator

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HIGHLIGHTS

- The subordinated stable CARMA process is examined.
- As a subordinator the stable process is considered.
- The codifference is considered as a measure of dependence of given process.
- The estimation and simulation procedures are presented.

ARTICLE INFO

Article history:
Received 13 May 2015
Received in revised form 3 July 2015
Available online 31 October 2015

Keywords:
Subordination
CARMA process
Ornstein–Uhlenbeck process
Codifference
Simulation
Estimation

ABSTRACT

In this paper we examine the continuous-time autoregressive moving average process driven by α-stable Lévy motion delayed by inverse stable subordinator. This process can be applied to high-frequency data with visible jumps and so-called "trapping-events". Those properties are often visible in financial time series but also in amorphous semiconductors, technical data describing the rotational speed of a machine working under various load regimes or data related to indoor air quality. We concentrate on the main characteristics of the examined subordinated process expressed in the language of the measures of dependence which are main tools used in statistical investigation of real data. However, because the analyzed system is based on the α-stable distribution therefore we cannot consider here the correlation (or covariance) as a main measure which indicates at the dependence inside the process. In the paper we examine the codifference, the more general measure of dependence defined for wide class of processes. Moreover we present the simulation procedure of the considered system and indicate how to estimate its parameters. The theoretical results we illustrate by the simulated data analysis.

1. Introduction

In the real data analysis the selection of appropriate model suitable to examined signal is the most important issue. The one of the easiest ways to find a proper model, constructed on the stochastic properties of examined data, is based on the assumption that the vector of observations contains realizations of independent random variables with the same distribution. However, in most time series the structure of dependence is more complicated and we observe strong dependence between examined data. Therefore there is need to analyze more sophisticated models in order to cover all properties of the signal. One of the simplest system which takes under consideration the dependence inside the examined

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http://dx.doi.org/10.1016/j.physa.2015.10.081
0378-4371/

process is the discrete-time autoregressive moving average (ARMA) time series [1]. This model has a simple structure therefore it can be used not only in the theoretical analysis, it also captures many interesting features of real data from different fields. Moreover in the recent years there were introduced many extensions of the classical version of ARMA model. We only mention here few of them, like introduction of time-dependent parameters instead of the constant ones or application of noise which has heavier-tail than in the Gaussian case (i.e. α-stable distribution) [2–5].

For many real data it is more adequate to specify model in continuous time rather than in discrete time [6]. The continuous-time systems seem to be the most natural for example for modeling high frequency data, like in finance. Moreover they are also more appropriate for irregularly spaced data or in case of missing observations. Therefore in recent years the continuous-time models have become more popular. One of the famous examples of continuous-time models is the Ornstein–Uhlenbeck process that was originally introduced by Uhlenbeck and Ornstein [7] as a proper model for the velocity process in the Brownian diffusion. In other words, this process provides a stationary solution for the classical Klein–Kramers dynamics, see Ref. [8] and reference therein. First of all the Ornstein–Uhlenbeck process has been used in applications to financial data such as interest rates, currency exchange rates, and commodity prices [8,9], but there are known another applications, for instance in physics where this system was a prototype of a noisy relaxation process. It is worth mentioning, the Ornstein–Uhlenbeck process is known in finance as the Vasiček model [10].

However, the classical Ornstein–Uhlenbeck process is only a special case of the continuous-time ARMA model, called CARMA system with Gaussian structure. The CARMA model is the solution of a higher order system of stochastic linear differential equations, which can be seen as linearly filtering of the random input. The discrete version of CARMA process is a discrete-time ARMA time series therefore there is a variety of its possible applications. In the last years the classical CARMA models based on the Brownian diffusion were analyzed in different aspects. We only mention some of them, like problem estimation for continuous-time moving average processes [11,12], limit behavior of periodogram of CARMA systems [13] or theory for multivariate CARMA models [14]. Similar as in the discrete-time case there are also considered extensions of the classical CARMA systems. In the literature the most popular extension is based on the replacement of the Brownian motion in the classical definition of CARMA model by the more general class of processes, namely belonging to the infinitely divisible class of distributions with particular attention on the α-stable Lévy motion [15–17]. It is worth mentioning, the CARMA models with α-stable distribution can be successfully applied for example to high-frequency data with observed jumps [18].

Unfortunately, in many cases models like continuous-time ARMA (even with different than Gaussian structure) are not sufficient to description of real data, especially with observable, so called “trapping events”. By “trapping events” we mean visible in the data time periods where the process stays on the same level. This effect we observe for example in financial data, especially interest rate data quoted for developing countries where the market conditions do not change so fast [8] but also for technical data describing rotational speed of the machine. Detection and parametrization of such events might help to improve efficiency of machine usage, for example to minimize number of segments and their duration for machine operation under idle mode or overload mode [19]. This special behavior can be also seen in the data related to the indoor air quality [20–22]. One of the model that can be used in this context is the co-called subordinated process in which the “normal” time is replaced by the another non-negative non-decreasing process, called subordinator. In the above mentioned examples such subordinator is constructed as an inverse process to another strictly increasing Lévy subordinator. More precisely, the mentioned subordinated process is a superposition of two independent systems, one — called the external or parent process and the second — called the operational time (or internal process) and given by the inverse subordinator. There are many examples of such construction. Let us mention for example the arithmetic Brownian motion with inverse tempered stable subordinator [21] or Brownian motion with inverse α-stable, tempered stable or Gamma subordinators [23]. The subordinated processes, among others, are examples of anomalous diffusion models and there are known connections between them and the fractional Fokker–Planck type equations [24,25]. More precisely, the probability density function of the subordinated processes is described by the fractional Fokker–Planck type equation that depends on the external process and distribution of the subordinator. For interesting examples see Refs. [26–29]. We should mention here that one of the most popular process used as the subordinator applied for different application is the α-stable one [8] however in the literature one can find the analysis of different systems that can be used as the internal processes in the construction of subordinated models, see for example Ref. [23].

It is worth to stress that processes delayed by inverse subordinators are continuously considered in physics since the pioneering work of Scher and Montroll [30]. In the last decade they become very popular especially from the practical point of view therefore there was a need to consider their properties that may be useful in the statistical investigation of real data. One of the characteristic that may help to fit a proper model to real data is the structure of dependence. The most popular measure of dependence is correlation (or covariance). By comparing the empirical counterparts of autocorrelation (or autocovariance) calculated for real data with the theoretical one for given theoretical model we can answer the question if the model is well fitted to examined vector of observations [31]. However, for the stochastic processes with heavy-tailed distributions for which the variance diverges, these tools are inadequate. Therefore there is need to consider another measures of dependence more adequate for analyzed processes. One of the measure that can be applied to variety of systems is the codifference [32] which is based on the characteristic function of examined process. Moreover, the codifference in the Gaussian case reduces to the classical covariance, so it can be treated as the natural extension of the well-known measure. On the other hand, according to the definition, it is easy to evaluate the empirical codifference which is based on the empirical characteristic function of the analyzed data. It is worth to mention that the codifference is closely related to the