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Maximum entropy spectral analysis for streamflow forecasting

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HIGHLIGHTS

- Configurational entropy spectral analysis is developed with spectral power as a random variable.
- The proposed spectral analysis yields Burg's maximum entropy spectral analysis.
- The maximum entropy spectral analysis encompasses the Burg entropy spectral analysis and two configurational entropy spectral analyses.

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ABSTRACT

Configurational entropy spectral analysis (CESAS) is developed with spectral power as a random variable for streamflow forecasting. It is found that the CESAS derived by maximizing the configurational entropy yields the same solution as by the Burg entropy spectral analysis (BESA). Comparison of forecasted streamflows by CESAS and BESA shows less than 0.001% difference between the two analyses and thus the two entropy spectral analyses are concluded to be identical. Thus, the Burg entropy spectral analysis and two configurational entropy spectral analyses form the maximum entropy spectral analysis.

1. Introduction

The entropy theory comprising the Shannon [1] entropy and the principle of maximum entropy (POME) [2,3] has been widely applied in hydrology [4–8]. The advantage of using the entropy theory is that it combines statistical information with physical characteristics and provides least-biased estimation. However, it was not used for forecasting until Burg [9,10] developed the maximum entropy spectral analysis (MESA) which is called the Burg entropy spectral analysis (BESA). The Burg entropy is defined in terms of frequency f as a random variable:

$$H_B(p) = \int_{-W}^{W} \ln[p(f)]\mathrm{d}f \tag{1}$$

where frequency f is considered as a random variable, W is the Nyquist frequency, and $p(f)$ is the normalized spectral density taken as the probability density function (PDF) of f.

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For a stationary random process BESA computes spectral power from the autocorrelation of given lags, without assuming autocorrelation of unknown lags as zero [11]. It has an advantage over classical method in terms of computational ease, short and smooth spectra with a high degree of resolution, and robustness of estimates and their stability. As a result, BESA has been widely applied to spectral analysis of geomagnetic fields, climate indices, surface air temperature, geophysical exploration, tide levels, precipitation, and runoff [12–21]. BESA has also been employed for long-term streamflow forecasting and real-time flood forecasting [22–25,6,7] and has been shown to have an advantage in long-term streamflow forecasting over traditional stochastic methods, but has not been found to be superior for short-term flow forecasting.

The maximum entropy spectral analysis can be derived using the configurational entropy introduced by Frieden [26] and Gull and Daniell [27], which is defined as

$$H_{CF}(f) = -\int_{-W}^{W} p(f)\ln[p(f)]\mathrm{d}f. \tag{2}$$

It is noted from Eq. (2) that the configurational entropy is defined in the same form as the Shannon entropy. The configurational entropy spectral analysis with frequency as a random variable (CESAF) is shown to be preferred over BESA for autoregressive moving average (ARMA) and moving average (MA) processes [28]. CESAF has been applied to monthly streamflow forecasting, and has been found to perform better than BESA [29].

On the other hand, configurational entropy spectral analysis can be derived with spectral power as a random variable (CESAS). The streamflow time series $y_t,\ t = 1, 2, \ldots, T$ can be transferred to spectral powers $x_k,\ k = 1, \ldots, n$, in the frequency domain by the Fast Fourier transform. For each frequency f_k there is one associated spectral power x_k. Let $\vec{x} = (x_1, x_2, \ldots, x_n)$ and let it be assumed that each probability density function $p\,(x_k)$ is considered independent identically distributed. Then the joint probability density function can be noted as $p(\vec{x}) = p(x_1)\cdots p(x_n)$. Now assuming each spectral power x_k as a random variable, the configurational entropy is defined as

$$H_{CS}(p) = -\int_{D} p(\vec{x})\ln[p(\vec{x})]\mathrm{d}\vec{x} = -E\{\ln[p(\vec{x})]\}. \tag{3}$$

However, it was shown by Gray [30] that if x_k came from an N-dimensional Gaussian distribution, then the joint distribution can be given by

$$p(\vec{x}) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}}\left(\frac{1}{\det\Re}\right)^{\frac{1}{2}}\exp\left(-\frac{1}{2}\vec{x}^t\Re^{-1}\vec{x}\right) \tag{4}$$

where $\det\Re$ is the determinant of the autocorrelation matrix defined by

$$\Re = E[Y^TY] = \begin{bmatrix} \rho_0 & \rho_1 & \cdots & \rho_{n-1} & \rho_n \\ \rho_1 & \rho_0 & \cdots & \rho_{n-2} & \rho_{n-1} \\ \vdots & & & & \vdots \\ \rho_{n-1} & \rho_{n-2} & \cdots & \rho_0 & \rho_1 \\ \rho_n & \rho_{n-1} & \cdots & \rho_1 & \rho_0 \end{bmatrix} \tag{5}$$

where ρ_n is the autocorrelation of the nth lag. [Define matrix Y.] Substitution of Eq. (4) into Eq. (3) yields

$$H_{CS}(p) = \ln[(2\pi e)^{\frac{N}{2}}(\det\Re)^{\frac{1}{2}}]. \tag{6}$$

It is noted that the autocorrelation is linked to the spectral density. Thus, replacing the autocorrelation in Eq. (6) with spectral density, the result is [give the intermediate steps] $H(f) = \int_{-W}^{W}\ln[p(f)]\,\mathrm{d}f$, which is the Burg entropy.

The objective of this paper therefore is to derive the configurational entropy spectral analysis with spectral power as a random variable, and to show how it yields Burg entropy spectral analysis.

2. Review of Burg entropy spectral analysis

Using the principle of maximum entropy, Burg [9,10] developed BESA for a stationary random process, which provides a basis to connect the spectra with the autoregressive (AR) process. By maximizing the Burg entropy in Eq. (1) with the use of the method of Lagrange multipliers, he obtained the spectral density as

$$P(f) = \frac{1}{\sum_{n=-N}^{N}\lambda_n e^{-i2\pi fn\Delta t}} \tag{7}$$