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Revised lattice Boltzmann model for traffic flow with equilibrium traffic pressure

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HIGHLIGHTS

- This study proposes a revised lattice Boltzmann model with traffic pressure.
- Establishing approach of traffic flow equilibrium velocity distribution is given.
- The expression of traffic pressure is put forward.
- Macroscopic dynamic characteristics of the new model are investigated.
- The effect of equilibrium traffic pressure on traffic flow stability are studied.

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ABSTRACT

A revised lattice Boltzmann model concerning the equilibrium traffic pressure is proposed in this study to tackle the phase transition phenomena of traffic flow system. The traditional lattice Boltzmann model has limitation to investigate the complex traffic phase transitions due to its difficulty for modeling the equilibrium velocity distribution. Concerning this drawback, the equilibrium traffic pressure is taken into account to derive the equilibrium velocity distribution in the revised lattice Boltzmann model. In the proposed model, a three-dimensional velocity-space is assumed to determine the equilibrium velocity distribution functions and an alternative, new derivative approach is introduced to deduct the macroscopic equations with the first-order accuracy level from the lattice Boltzmann model. Based on the linear stability theory, the stability conditions of the corresponding macroscopic equations can be obtained. The outputs indicate that the stability curve is divided into three regions, i.e., the stable region, the neutral stability region, and the unstable region. In the stable region, small disturbance appears in the initial uniform flow and will vanish after long term evolution, while in the unstable region, the disturbance will be enlarged and finally leads to the traffic system entering the congested state. In the neutral stability region, small disturbance does not vanish with time and maintains its amplitude in the traffic system. Conclusively, the stability of traffic system is found to

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be enhanced as the equilibrium traffic pressure increases. Finally, the numerical outputs of the proposed model are found to be consistent with the recognized, theoretical results.

1. Introduction

Traffic flow system presents significant impact on social economic and urban environment [1,2]. In the past decades, many traffic flow models have been developed to reproduce all kinds of traffic jam phenomena, but no perfect model achieved yet [3].

The system of traffic is open and giant complex, the modeling thought based on lattices present certain advantages for its convenience, such as lattice hydrodynamic models [4,5] in macroscopic level and cellular automaton models in microscopic level [6,7]. For example, introducing revolutionary games [8,9] to cellular automaton Biham–Middleton–Levine (BML) model [7] results in a premature occurrence of traffic jams and thus unnecessarily burdens the transportation system.

In this paper, the study focus on the lattice Boltzmann traffic model, which is one of kinetic-type models in mesoscopic level. The drawbacks of the kinetic-type models are identified [10]. Despite the presumed assumptions such as vehicle chaos assumption, there are still number of unknown parameters and empirical relations need to be estimated via observations. Furthermore, the mathematically integral–differential-type equations are usually difficult to be solved with either numerical methods or analytical methods.

Meng et al. [10] introduced the Bhatnagar–Gross–Krook-type approximation to Boltzmann equation and proposed a lattice Boltzmann model to study traffic phenomena. The model can simulate traffic flow efficiently and provide certain meaningful results explaining the physical phenomenon. However, seeking relevant equilibrium velocity distribution function is still a challenging task as no momentum and energy conservations exists in traffic flow. Hence, in this study, a revised lattice Boltzmann model, considering the traffic pressure and the velocity–density relation, is proposed.

2. Model development

2.1. Lattice Boltzmann model

The governing equation of lattice Boltzmann model for traffic flow, proposed by Meng et al. [10], is given as below:

$$f_i\left(x+v_i\delta_t, t+\delta_t\right)-f_i\left(x,t\right)=\omega\left[f_i^{eq}\left(x,t\right)-f_i\left(x,t\right)\right] \tag{1}$$

where $f_i\left(x,t\right)$ is the phase-space density, it denotes the distribution of vehicles moving with velocity v_i at location x and time t. f_i^{eq} is the equilibrium phase-space density, namely, the distribution of vehicles under the local equilibrium state, which describes the local equilibrium state resulting from the competition between two opposite sides in a local region, i.e., the drivers' effort toward their desired speeds and the interactions with other vehicles, namely, the equilibrium phase-space density. ω the dimensionless relaxation factor, $\omega \equiv \delta_t/\tau$, δ_t a small time clearance, and τ the relaxation time. Similar to fluid and granular media, it is known that the phase-space density also depends on their gradients, which is a consequence of the finite adaptation time required to reach local equilibrium.

In hydrodynamics, one can derive the appropriate equilibrium distribution function f_i^{eq} according to the discretization method of phase space. But such function usually contains certain unknown coefficients, which need to be determined based on the conservation laws of fluid mechanics. However, there is no such conservation principle in traffic flow system, which causes dilemma in most traffic studies. Fortunately, it is possible to determine f_i^{eq} through the empirical observation under certain assumptions, similar to the velocity–density relation in the Lighthill–Whitham–Richards macroscopic model [11] for closing the mathematical formation.

By empirical and/or field measurements, the velocity distributions of traffic flow with a small fraction of trucks follow the Gaussian distribution [12,13]. However, if the sampling intervals are too large, the bimodal distribution may be observed [14,15], which reflect the transition from free to congested traffic. Helbing [3] summarized some empirical data from the literature [12–17] and stated that vehicle velocities are more or less Gaussian distributed. Hence, establishing proper velocity distribution function plays a key role in traffic flow modeling. A specific equilibrium phase-space density function [5] is given as follows,

$$f^{eq}=\rho\exp\left\{-\left(v-V_{eq}\right)^2/\left(2\theta_{eq}\right)\right\}/\left(2\pi\theta_{eq}\right)^{1/2} \tag{2}$$

where V_{eq} is the equilibrium average velocity, θ_{eq} is the equilibrium velocity variance, and $\theta_{eq}=A\left(\rho\right)V_{eq}^2$, $A\left(\rho\right)$ is the density-dependent variance prefactor, its empirical form proposed by Shvetsov and Helbing [18] is complicated. It is difficult to extend these functions to more complex traffic cases or numerical simulation.