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Symmetric air-spaced cantilevers for strain sensing

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ABSTRACT

Large mechanical strain is desirable for many devices such as cantilever-based accelerometers and vibration energy harvesting devices. Air-spaced cantilevers are proposed to significantly increase the mechanical strain experienced by the cantilever beams. In this paper, an analytical model for the bending of symmetric air-spaced cantilevers is established by decomposing the cantilever deformation into pure bending and S-shape bending. This model was further verified by a discrete component model. A criterion to determine the dominant bending mode is presented. Finite element simulations and experiments were also conducted and showed a good agreement with the analytical model.

1. Introduction

Cantilever structures have been widely used in MEMS. For instance, Fig. 1(a) shows the schematic structure of a cantilever-based piezoresistive or piezoelectric accelerometer [1–7]. When acceleration is applied, the proof mass displaces due to the inertial force and subsequently bends the cantilever. The neutral plane, where the strain is zero, is in the middle for a homogeneous cantilever with a rectangular cross-section. A sensing layer (piezoresistive or piezoelectric) can be integrated on the cantilever to measure the strain and hence the acceleration. Since the strain experienced is proportional to the distance between the sensing layer and the neutral plane, the sensing layer is usually fabricated on the top or bottom surface. Such cantilever-mass structure is also widely used for vibration energy harvesting if a piezoelectric layer is attached on the cantilever surface or the cantilever is made of piezoelectric bimorph as shown in Fig. 1(b) [8–13]. Similarly, the voltage generated due to piezoelectricity is proportional to the strain experienced by the piezoelectric material. The AC voltage generated by the mechanical vibration usually needs to be converted to DC voltage by a diode bridge. If the amplitude of the AC voltage is not large enough, only a small portion or none of the electrical energy generated by the piezoelectric material can be utilized by the following circuits. Therefore, it is also desirable to increase the distance between the piezoelectric layer and the neutral plane to obtain a large voltage.

Air-spaced cantilever has been proposed to significantly increase the accelerometer sensitivity and output voltage of vibration energy harvesting devices [14,15]. In this paper, a systematic study of symmetric air-spaced cantilevers is presented. An analytical model based on the decomposition of the cantilever bending is first introduced. Based on this model, formulas for spring constant, resonant frequency, average normal strain, shape function, and design criterion to select the right bending mode are derived. This model is then verified by discrete component model, FEM simulation, and experiments.

2. Analytical model

The schematic diagram of a symmetric air-spaced cantilever with a proof mass is shown in Fig. 2. The top and bottom layers of the air-spaced cantilever have the same dimension: $w \times t \times l$. The dimension of the proof mass is $w_{pm} \times t_{pm} \times l_{pm}$. The structure is symmetric along $y = w/2$ plane, and we only consider the deformation in the x–z plane (no torsion). It can be easily observed that the neutral plane of the composite cantilever is in the middle due to its symmetry in z direction. The distance between the geometric middle plane of the individual beams and neutral plane is αt. Since $w_{pm} \geq w$, interface force between the proof mass and beams is uniform in y direction.

When acceleration is applied to the proof mass in vertical direction, the proof mass will displace accordingly and the beams are subjected to deformation. The fundamental beam theory is developed based on the plane assumption, namely the transverse section of the beam remain plane after bending [16]. However, for the air-spaced cantilever, the plane assumption is not necessarily valid.

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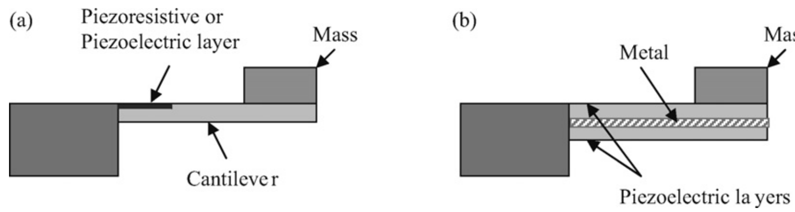

Fig. 1. (a) Schematic of a cantilever-type piezoresistive or piezoelectric accelerometer and (b) schematic of a vibration energy harvesting device based on piezoelectric bimorph.

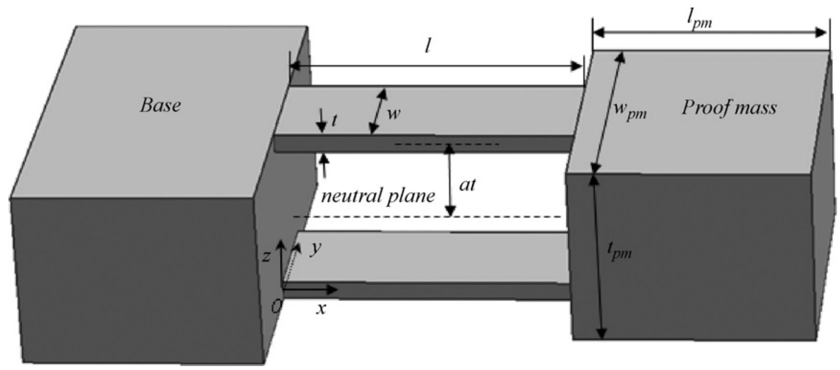

Fig. 2. Schematic structure of the air-spaced cantilever.

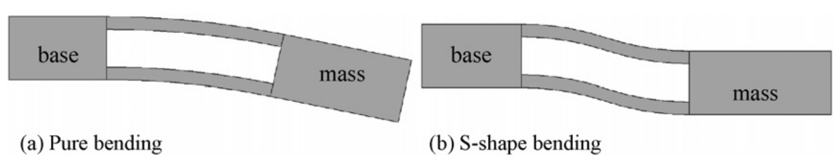

Fig. 3. Two deformation modes of the composite air-spaced cantilever.

Therefore, it is necessary to develop a new model to describe this kind of structure, facilitating its applications to accelerometers, vibration energy harvesting devices, cantilever probes, etc.

In practice, we usually can observe two deformation modes of the air-spaced cantilever. The first mode is the pure bending of the composite air-spaced cantilever as shown in Fig. 3(a). The second one is S-shape bending as shown in Fig. 3(b). Which mode is dominant mainly depends on the geometry parameters such as l, l_{pm}, t, and t_{pm}.

The decomposition of the deformation into two modes can be justified using the shear force and bending moment diagrams. When acceleration a is applied, the total shear force and bending moment experienced by the composite beam are presented in Fig. 4(a). As we can clearly observe in the figure, shear force is constant and the bending moment is a linear function of the horizontal position. If we note that (1) for pure bending, the composite beam are only subject to a constant bending moment without any shear force, and (2) for S-shape bending, the proof mass will remain horizontal after deformation (meaning the bending moment at the middle of the beam is zero), it is very easy to decompose the total shear force and bending moment to pure bending and S-shape bending modes as shown in Fig. 4(b) and (c). Note that we use sub-

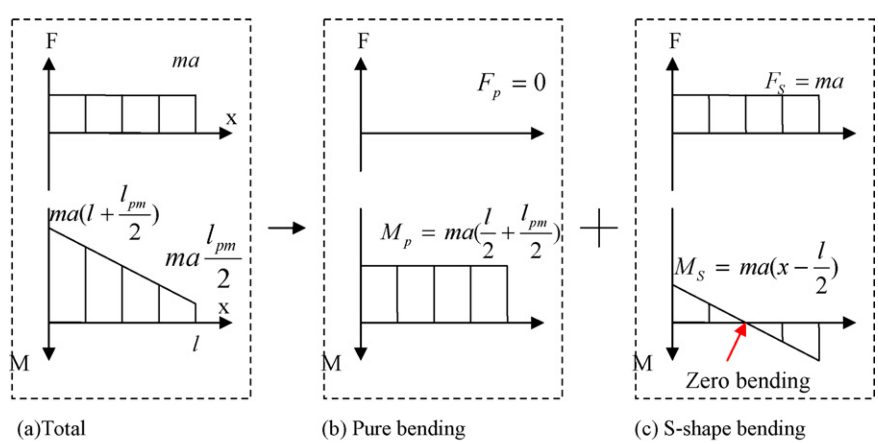

Fig. 4. Diagram of shearing forces and bending moments for the beam part. m is the mass of the proof mass and a is the acceleration applied. The mass of cantilever is neglected.