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Traffic jam at adjustable tollgates controlled by line length

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ABSTRACT

We present the stochastic model for the jam formation at the tollgates of which the number is adjusted by synchronizing with the jam's length. We study the jam formation and its fluctuation in front of the adjustable tollgates on a highway. Controlling the number of tollgates has an important effect on the jam formation. The jams are classified into three kinds: (a) localized jam, (b) synchronized jam, and (c) growing jam. The jamming transitions from the localized jam, through the synchronized jam, to the growing jam occur with increasing inflow probability. At an intermediate inflow, the jam fluctuates largely by synchronizing with the number of tollgates. When the inflow probability is higher than the sum of outflow probabilities at tollgates, the jam continues to grow and diverge with time. The dependence of the fluctuating jam on the inflow probability is clarified.

1. Introduction

Recently, the simplified traffic models have been proposed to clarify the cause and effect of jams. The physical concepts and methods have been applied to transportation systems [1–7]. Several physical models have been presented to the vehicular flow [8–43]. The jamming transitions between distinct traffic states have been studied from a point of view of statistical physics and nonlinear dynamics.

Traffic jams are a typical signature of the complex behavior of traffic flow. Traffic jams are classified into two kinds of jams: (1) spontaneous jam (or phantom jam) which propagates backward as the stop- and go-wave and (2) stationary jam which is induced by slowdown or blockage at a section of roadway. In real traffic on toll roads, the jam of vehicles is induced just in front of tollbooths because the delay occurs by collecting tolls. The tollgate works as a bottleneck. In order to reduce the delay or congestion, more tollbooths have been introduced into the toll roads. The jam at a tollgate is reduced by increasing the tollgates. The jam formation at the tollgates has been studied by the use of the car-following and stochastic models [44–46].

The traffic jam at the tollgates is closely connected to the queueing process. The stationary queueing processes have been studied extensively by the use of stochastic methods. For multiple service windows, there are two types of queueing systems: forke-type and parallel-type systems [47,48]. The queueing processes are stationary. In the stationary queue, the assumption of detailed balance holds. The analytic solution has been derived from the detailed balance [47,48]. For the multiple service windows, when customers decrease, some windows not working are not necessary until customers increase. In order to avoid the uselessness, the cash resisters in super markets increase or decrease by synchronizing with the number of customers. The queueing process is not stationary but non-stationary because the queue length varies greatly with the number of cash resisters. The queueing process is a time-dependent dynamic system. The non-stationary (time-dependent) queueing problem has little been studied.

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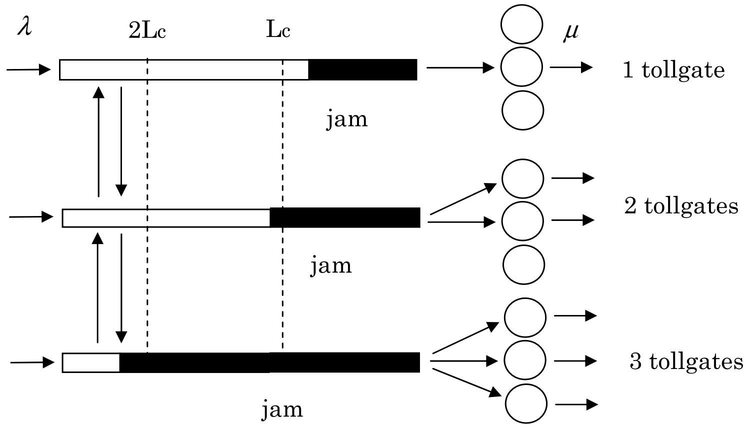

Fig. 1. Schematic illustration of the stochastic model for the jam formation in the case of three adjustable tollgates. The number of tollgates is adjusted by synchronizing with the jam length. A vehicle flows into the jam with probability λ. The vehicle in the front of each tollgate flows out of each tollgate with probability μ independently.

The time-dependent forke-type queueing system is applied to the tollgate system on a highway. It is necessary and important to study the dynamic behavior of traffic jam at the tollgates of which the number is adjusted by synchronizing with the jam's length. However, the jam formation at the adjustable tollgates has little been studied. Also, the fluctuation of inflow and outflow is important. It is necessary and important to take into account the fluctuating inflow and outflow.

In this paper, we propose the stochastic model for the jam formation at the adjustable tollgates to account of the fluctuating inflow and outflow. We study the jam formation and its fluctuation in front of the adjustable tollgates. We compare the jam formation at the adjustable tollgates with that at the normal tollgates. We clarify the effect of adjustable tollgates on the jam and its fluctuation.

2. Model

We consider the jam of vehicles in front of adjustable tollgates on a toll highway. Vehicles flow into the jam at adjustable tollgates with fluctuating rate. When the jam (line) length is less than the critical length L_c, the first tollgate works. If the line length is superior to the critical length, the second tollgate is added to the system and the first and second tollgates work simultaneously. When the line length is superior to S times critical length, $(S + 1)$ tollgates work simultaneously. The number of tollgates varies by synchronizing with the line length. Vehicles lining up at tollgates flow out of each tollgate with fluctuating rate. The jam is formed in the front of tollgates. Also, the jam fluctuates with stochastic inflow and outflow. The jam continues to vary with time. We mimic the jam formation of vehicles in front of adjustable tollgates on a highway. We consider the vehicular flow under the open boundary condition.

Fig. 1 shows the schematic illustration of the stochastic model for the jam formation in the case of three adjustable tollgates. A vehicle flows into the jam with probability λ. If only the first tollgate works, the vehicle at the first tollgate flows out of the tollgate with probability μ. When the first and second tollgates work, vehicles at the first and second tollgates flow out of the first and second tollgates independently with probability μ. If S tollgates work simultaneously, vehicles flow out of each tollgate independently.

The number of vehicles within the jam at the adjustable tollgates is defined as $N(t)$ at time t. The jam length at the tollgates is given by $N(t)\Delta d$ where Δd is the mean distance between vehicles within the jam. If the jam length is superior to the critical value, the first and second tollgates work. When the jam length is superior to S times critical value, $S + 1$ tollgates work simultaneously.

In the case of two tollgates, the number $N(t + 1)$ of vehicles within the jam at the adjustable tollgates at time $t + 1$ is given by

$$N(t+1) = N(t) + \xi(t) - \theta(N(t) - 1)\{\zeta_1(t) + \theta[N(t) - L_c/\Delta d]\zeta_2(t)\}, \tag{1}$$

where $\xi(t)$, $\zeta_1(t)$, and $\zeta_2(t)$ are independent Boolean random variables. $\xi(t)$ represents the inflow rate at time t. $\xi(t) = 1$ with probability λ and 0 with probability $1 - \lambda$. $\zeta_1(t)$ represents the outflow rate from the first tollgate at time t. $\zeta_1(t) = 1$ with probability μ and 0 with probability $1-\mu$. $\zeta_2(t)$ represents the outflow rate from the second tollgate at time t. $\zeta_2(t) = 1$ with probability μ and 0 with probability $1 - \mu$. $\theta(t)$ is the step function ($\theta(x) = 1$ if $x \geq 0$ and 0 if $x < 0$). The second term on the right hand side represents the inflow. The third term on the right hand side represents the outflow from the first tollgate. The fourth term on the right hand side represents the outflow from the second tollgate. If there are no vehicles in front of the tollgates, the outflow out of the tollgates does not occur. This outflow is represented by the step function of the third term on the right hand side. Thus, the time evolution of the number of vehicles within the jam at two adjustable tollgates is described by Eq. (1).