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Optimal weight based on energy imbalance and utility maximization

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HIGHLIGHTS

- Combine energy imbalance model with utility maximization.
- Obtain optimal weight for both genders when maximizing lifetime utility.
- Investigate trajectory to steady state weight.
- Numerical example to illustrate the relationship between food consumption and weight across both genders.

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ABSTRACT

This paper investigates the optimal weight for both male and female using energy imbalance and utility maximization. Based on the difference of energy intake and expenditure, we develop a state equation that reveals the weight gain from this energy gap. We construct an objective function considering food consumption, eating habits and survival rate to measure utility. Through applying mathematical tools from optimal control methods and qualitative theory of differential equations, we obtain some results. For both male and female, the optimal weight is larger than the physiologically optimal weight calculated by the Body Mass Index (BMI). We also study the corresponding trajectories to steady state weight respectively. Depending on the value of a few parameters, the steady state can either be a saddle point with a monotonic trajectory or a focus with dampened oscillations.

1. Introduction

The epidemic of obesity has spread at an alarming speed in recent decades. Currently, 35.7% of the adult population in America are suffering from obesity based on a study from the Centers for Disease Control and Prevention [1]. Obesity can be very harmful since it reduces one's life expectancy and raises the probability of many diseases such as heart attack, type 2 diabetes and hypertension [2,3].

In order to control the obesity epidemic, it is crucial that we understand how weight changes. Scholars have used various models to search for factors that impact weight [4–10]. They have found out that energy imbalance plays a big role in weight motion [6,11,12]. Energy imbalance model describes an individual's weight gain or loss based on differences between energy intake and expenditure [13]. Excess energy intake can easily lead to an excess of weight. The increasing consumption of calorie dense food such as fast food hugely raises the amount of energy input [14]. At the same time, modern life style requires less physical activity thus decreases energy expenditure [15]. Consequently, this combination of increased energy intake and reduced energy expenditure predisposes individuals to weight gain and obesity.

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While energy imbalance identifies how individuals change weight, it does not answer why individuals may be compelled to eat more or move less. Many debated hypotheses have been proposed to explain this phenomenon [16–18]. Some of these include socioeconomic factors [19,20], environmental influences on lifestyle [21], and potential impact of comorbidities on weight gain [22]. In order to quantify the benefits individuals obtain by changing food consumption or physical activity, economists introduced the notion of utility function [16,23]. Based on previous models proposed by Levy and Dragone [24,25], we hypothesize that individuals make decisions to maximize their lifetime utility. In addition, optimal control method can be useful in analyzing models. It has been used in many previous studies to solve problems such as vaccination strategy [26], intervention design [7], or disease control [27]. Here we adopt this method to derive an optimal weight that maximizes lifetime utility.

In this paper, we combine an energy imbalance model with a lifetime utility function to analyze weight changes. Through applying well-developed mathematical tools from optimal control theory, we first obtain the degree of energy intake that yields an optimal physiological weight. Then we calculate the optimal weight by maximizing a derived lifetime utility function. By showing that the derived optimal weight is larger than the physiologically optimal weight, this study provides an explanation for the prevalence of obesity. In addition, we show the corresponding trajectories to the optimal weight for both genders. To the best of our knowledge, few previous studies have studied utility maximization and trajectories to steady state weight simultaneously. The organization of the paper is as follows. In Section 2 we introduce the energy imbalance model for both genders. Section 3 derives the optimal weight for male and Section 4 derives that for female. In the last section, we first offer a figure to illustrate food consumption across different steady state weight for both genders, then give a brief discussion of the study.

2. Model description

The model in this study contains two main components. First, we develop a state equation to describe weight motion from differences in energy intake and expenditure. The state equation is built upon existing energy imbalance models and includes the continual effects of changing utility and eating behavior [25]. The second component is a utility function that takes food consumption, eating habits and health status into consideration. An optimal weight that maximizes lifetime utility is then derived based on the two components.

Let FFM be the total kg of fat free mass at time t and F be the total kg of fat mass of the body at time t. We know that $\omega = FFM + F = \alpha_1 F + b + F = (\alpha_1 + 1)F + b$, where b is the baseline data and it is positive [28]. We also know α_1 is different for both genders. Here we estimate α_{m1}, α_1 for male, to be 0.56; and α_{f1}, α_1 for female, to be 0.32 according to previous studies [28].

We can obtain a model of energy intake and expenditure: $c_f \frac{\mathrm{d}FFM}{\mathrm{d}t} + c_l \frac{\mathrm{d}F}{\mathrm{d}t} = I - E$, where I stands for the rate of energy intake, E is the rate of energy expenditure, c_f and c_l denote the energy density of FFM and F respectively [28]. It is clear from the above model that the change of body mass is determined by the difference of energy intake and expenditure. Next we present some estimated values from experiments. The approximated value for c_f is 7165, for c_l is 955.384 [29]. We further derive an explicit expression of E [29].

$$E_F = 0.0278\omega^2 + 9.2893\omega + 1528.9$$

for female and,

$$E_M = -0.0971\omega^2 + 40.853\omega + 323.59$$

for male. Since $\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}FFM}{\mathrm{d}t} + \frac{\mathrm{d}F}{\mathrm{d}t} = (\alpha_1 + 1)\frac{\mathrm{d}F}{\mathrm{d}t}$ and $\frac{\mathrm{d}FFM}{\mathrm{d}t} = \alpha_1 \frac{\mathrm{d}F}{\mathrm{d}t}$, we have $(c_f\alpha_1 + c_l)\frac{\mathrm{d}F}{\mathrm{d}t} = I - E$. And in turn we get $\frac{\mathrm{d}\omega}{\mathrm{d}t} = (\alpha_1 + 1)\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\alpha_1+1}{c_f\alpha_1+c_l}(I - E)$.

Denote an eating model where rational people maximize their expected lifetime utility. Such a utility is consisted of three parts: positive utility coming from food consumption, negative utility of changing eating habits and survival rate based on health conditions. $c(t)$ is the total amount of food consumption at time t, so we write positive utility of food consumption as $U(c(t))$. Since an increase of consumption brings positive utility, $U_c > 0$. Due to a diminishing marginal return, we have $U_{cc} < 0$. Negative utility of changing eating habits is defined as $-a\frac{\dot{c}(t)^2}{2}$, where a is a positive constant depicting the marginal disutility of changing eating habits. $\Phi((\omega(t) - \omega^*)^2)$ is the survival function measuring the probability of a person to live beyond time t. ω^* is the physiologically optimal weight according to Body Mass Index (BMI). We assume a deviation from the physiologically optimal weight lowers one's survival rate, that is $\Phi'(\cdot) \triangleq \frac{\partial\Phi((\omega(t)-\omega^*))^2}{\partial(\omega(t)-\omega^*)^2} < 0$. We let the probability of survival be concave as well, which implies $\Phi_\omega = \frac{\partial\Phi(\cdot)}{\partial\omega}$ and $\Phi_{\omega\omega} = \frac{\partial\Phi^2(\cdot)}{\partial^2\omega}$, this also requires $\Phi_{\omega\omega} < 0$ for all ω in the relevant domain.

3. Energy imbalance and optimal weight for adult male

Because there are a few gender-dependent variables, we study the case for male and female separately. First we analyze the weight motion model for adult males. The corresponding model is $\dot{\omega} = \frac{\alpha_{m1}+1}{c_f\alpha_{m1}+c_l}(I + 0.0971\omega^2 - 40.853\omega - 323.59)$.