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Structural diversity effects of multilayer networks on the threshold of interacting epidemics

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HIGHLIGHTS

- We introduce the "top–bottom" framework to define multilayer networks.
- We use the framework to solve collaboration–competition coexisting epidemic model.
- We introduce three diversity indicators, i.e. richness, evenness, and likeness.
- Both level and type of network diversity affect the epidemic dynamics.
- Transmission and collaboration are trade-off in diverse multilayer networks.

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ABSTRACT

Foodborne diseases always spread through multiple vectors (e.g. fresh vegetables and fruits) and reveal that multilayer network could spread fatal pathogen with complex interactions. In this paper, first, we use a "top-down analysis framework that depends on only two distributions to describe a random multilayer network with any number of layers. These two distributions are the overlaid degree distribution and the edge-type distribution of the multilayer network. Second, based on the two distributions, we adopt three indicators of multilayer network diversity to measure the correlation between network layers, including network richness, likeness, and evenness. The network richness is the number of layers forming the multilayer network. The network likeness is the degree of different layers sharing the same edge. The network evenness is the variance of the number of edges in every layer. Third, based on a simple epidemic model, we analyze the influence of network diversity on the threshold of interacting epidemics with the coexistence of collaboration and competition. Our work extends the "top-down" analysis framework to deal with the more complex epidemic situation and more diversity indicators and quantifies the trade-off between thresholds of inter-layer collaboration and intra-layer transmission.

1. Introduction

Over the last decade, the use of the network has been proved to be a powerful approach to model the structural complexity of complex systems [1–5]. A large body of theoretical literature discusses how network structures may shape the spread of infectious diseases and influence the design of optimal control strategies [6–8]. Such works usually focus on the single network and isolated epidemics. In most of the real-world complex systems, however, nodes in the system

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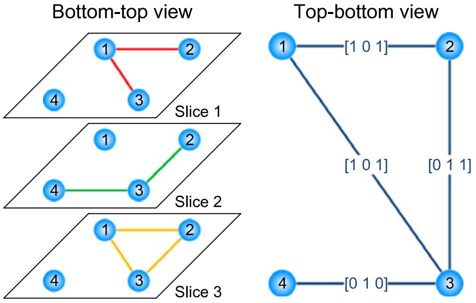

Fig. 1. Comparison of "bottom-up" and "top-down" frameworks, and illustration of network diversity. In the bottom-up framework, the multilayer network is described by the degree distribution of each layer. The degree distribution of layer 1 is $p_1(0) = 0.25$; $p_1(1) = 0.5$; $p_1(2) = 0.25$, that of layer 2 is $p_1(0) = 0.25$; $p_1(1) = 0.5$; $p_1(2) = 0.25$, and that of layer 3 is $p_3(0) = 0.25$; $p_3(2) = 0.75$. In the top-down framework, the multilayer network is described by an overlaid degree distribution [$p(1) = 0.25$; $p(2) = 0.5$; $p(3) = 0.25$] and an edge-type distribution [$q([010]) = 0.25$; $q([011]) = 0.25$; $q([101]) = 0.5$]. Furthermore, the richness of the multilayer network is 3; the 'likeness' is 1.75 (because there are three overlaid edges constructed from two intra-layer edges and one overlaid edge constructed from one individual edge), and the evenness is 0.98 (because there are in total seven intra-layer edges, with layers 1 and 2 containing two edges each and layer 3 containing three edges).

can engage in multiple interactions or edges. For example, in the global trade system, countries interact via various trade channels ranging from agricultural products to electronic products [1]. Additionally, in society people interact via their friendships, family relationships and/or more formal work-related links [9].

The recent infection of enterohemorrhagic Escherichia coli (EHEC) in Europe and Listeria monocytogenes in the United States poses a new challenge to the single network approach. The unique network for several reasons is not an appropriate one to explore such diseases mediated by trade networks. First, the trade-mediated epidemics involve multiple vectors to spread single viral agent, and each vector could form a particular network layer to link the same set of countries and territories [6,7,10,11]. Second, the viral agent involves complex interactions between multiple vectors, including both collaborating interaction [12–14] and competitive interaction [6,15,16]. With collaborating interaction, infection in one network layer can cause infecting processes on other layers within a node. Contrarily, competitive interaction (e.g. leaky partial immunity) leads to infection from one network layer reducing transmitting probability from or to other layers. These new features and uncertainties affect our ability to assess the global behavior of trade-mediated diseases and control their diffusion.

Many studies have focused on diffusion dynamics on multilayer networks with two layers, because of an initial interest in interdependence or overlay of two network layers [6,7,10]. However, it is notable that only a few previous studies of diffusion dynamics in multilayer networks study more than two types of edge [8] and understanding of the generic effects of diversity on diffusion behavior remains unclear. Recently, Paczuski et al. [8] considered percolation on interdependent locally treelike networks and the method can be conveniently extended to any number of interdependent networks. A serial of papers has studied multiple types of diseases competitively spread over one or more networks [17]. These studies have provided pioneering insights into multilayer networks, showing that the presence of more than one type of edge, or channel of interactions, in a network can lead to nontrivial effects.

In previous studies, although the specific contexts are various, we can regard each type of edge as a network layer [18], and then these studies can be described by a uniform framework at the mathematical level. A multilayer network can be represented by the overlay of multiple network layers (Fig. 1), with a set of r network layers. Each layer connects the same number N of nodes with their set of edges. Each infectious agent is represented by one node in all r network layers while its neighbors are independent for each of the layers. Researchers have tended to adopt the "bottom-up" framework for multilayer networks [18]. Here, "bottom" means the properties of each network layer, and "up" means the integration of these properties to describe and analyze the characteristics of entire multilayer networks. Usually, a multilayer network is defined by the degree distribution of each network layer and the correlation between all pairs of degree distributions. In our previous work [19], we present a different framework to study multilayer networks, i.e. the "top-down" analysis framework. The framework directly uses a few overall properties to define multilayer networks, and we can deduce the properties of each network layers from the definition if necessary. In this work, we have used the framework to reveal the effect of collaborating patterns on simple interaction epidemics on multilayer networks. However, the ability of the "top-down" should be evaluated in the more complex situation, especially with more interaction types.

One of the most key structural properties of multilayer networks is the relationship between layers, and degree distribution correlation is the major metric to the relationship. However, the degree correlation is restricted to deal with two network layers. Therefore, a general model and solution of epidemic dynamics in multilayer networks with more than two layers remain rare. Additionally, structural diversity has been recognized as another critical factor in control network dynamics [20–24]. However, the study of the diversity of multiple network layers is still in their initial stage. In our previous work [19],we use evenness and likeness of two network layers to evaluate the effect of diversity to collaborating epidemics. However, the paper still do not consider the effect of richness (i.e. the number of network layers) to interacting epidemics.

In this paper, we extend the "top-down" analysis framework to epidemic model with the coexistence of collaborating and competitive interactions. We also consider three indicators to measure the different types of "network diversity" including