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Minimum structural controllability problems of complex networks

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HIGHLIGHTS

- In some networks, the minimum driver vertex set cannot ensure the full control.
- Controllability is related to inaccessible strongly connected components.
- We propose an algorithm to identify minimum controlled vertex set.
- A mathematical model is established to identify minimum controlled vertex set.

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ABSTRACT

Controllability of complex networks has been one of the attractive research areas for both network and control community, and has yielded many promising and significant results in minimum inputs and minimum driver vertices. However, few studies have been devoted to studying the minimum controlled vertex set through which control over the network with arbitrary structure can be achieved. In this paper, we prove that the minimum driver vertices driven by different inputs are not sufficient to ensure the full control of the network when the associated graph contains the inaccessible strongly connected component which has perfect matching and propose an algorithm to identify a minimum controlled vertex set for network with arbitrary structure using convenient graph and mathematical tools. And the simulation results show that the controllability of network is correlated to the number of inaccessible strongly connected components which have perfect matching and these results promote us to better understand the relationship between the network's structural characteristics and its control.

1. Introduction

Recently the study of controllability of complex networks has been one of the attractive research areas for both literature of nature and engineering [1–8]. Controllability is an important property of control systems and plays a critical role in many control theory problems. According to control theory, a system is controllable if it can be driven from any initial state to any desired final state within finite time by suitable choice of inputs [9]. Structural theory has provided an efficient framework to study the controllability for an equivalent class of systems, where properties are explored based on the location of zero and non-zero values. In this paper we address the problem of identifying the minimum controlled vertex set for the controllability of these structural systems.

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The studies of minimum inputs and minimum driver vertices have received many advance results [10–18]. Commault et al. [10] proved the number of minimum inputs required to achieve structural controllability is equal to the number of minimum right-unmatched vertices. Liu et al. [11] proposed a method to identify the minimum driver vertex set *MDS* which consists of all the unmatched vertices of a maximal matching. It is obvious that the number of minimum driver vertices is equal to the number of minimum inputs. Wang et al. [12] optimized the minimum driver vertices making the minimum perturbation to the network. In Ref. [13], Menichetti et al. showed that the density of vertices which have 0, 1 and 2 in-degree and out-degree determines the number of driver vertices of random networks and proposed an algorithm to improve the controllability of networks. Ruths et al. [19] showed the fundamental structures that explain the basis of the correlation between topology of a network and certain control properties. Wu et al. [8] studied the problem of identifying a minimum steering vertex set that, if driven by different inputs, can achieve the transition of system between two specific states and propose an algorithm to identify the minimum steering vertex set. Moreover, Wu [8] showed that the size of minimal steering vertex set required for transition is much less than that for complete controllability.

Although the studies of minimum input and minimum driver vertices have obtained the above outstanding results, when the system digraph consists of multiple strongly connected components, Pequito et al. [20] showed the minimum driver vertices driven by different inputs may not ensure the controllability of the system. Especially, in this paper we demonstrate that the network cannot achieve the full control by driving minimum driver vertices when the associated graph contains inaccessible strongly connected components which have perfect matching.

Motivated by this problem, in this paper we focus on the problem of identifying a set of vertices that, if driven by inputs, can ensure the full control of the network. These vertices driven by inputs are called controlled vertices and we are particularly interested in identifying the minimum controlled vertex set *MCS* required to achieve structural controllability. Note that, the problem of identifying the minimum controlled vertex set is more complex than the problem of identifying the minimum inputs or minimum driver vertices. In Ref. [21], Olshevsky studied this problem for a given network with link-weight and proved that the problem of finding a minimum vertex set through which the system control can be achieved is NP-hard. Moreover, Guilherme et al. [22] proved the problem of finding the sparsest input matrix that ensures systems controllability is NP-complete by reducing this problem to a set covering problem. For a change, we study this problem for structural systems. Using strongly graph and mathematical tools, we map this problem into combination optimization with constraints.

The contribution of this paper is twofold. First, we demonstrate a direct correlation between the network's controllability and the number of inaccessible strongly connected components which have perfect matching. Second, we provide an algorithm to identify the minimum controlled vertex set through which control over networks with arbitrary structures can be achieved.

2. The structural controllability of complex network

In this paper, we consider a directed controlled network of N vertices and M inputs whose time evolution follows the continuous-time linear system

$$\dot{x} = Ax + Bu \tag{1}$$

where $x = (x_1, x_2, \ldots, x_N)^T$ stands for the state vector of N vertices; $u = (u_1, u_2, \ldots, u_M)^T$ is the input vector of M inputs; $A \in R^{N\times N}$ denotes the state matrix which describes the system's wiring diagram and the element a_{ij} represents the weight that vertex x_j can affect vertex x_i, $i, j = 1, \ldots, N$. $B \in R^{N\times M}$ is the input matrix which identifies the vertices which are driven by M inputs and the element b_{ij} represents the control strength from input u_j to vertex x_i, $i = 1, \ldots, N$, $j = 1, 2, \ldots, M$. But in most real networks the elements of A and B are often either unknown or known approximately except the zeros that mark the absence of connections between vertices [23]. Hence A and B are often considered to be structured matrices, i.e. their elements are either zeros or independent free parameters. Let $G(A, B) = (V, E)$ denote the graph of controlled directed network, where $V = V_A \bigcup V_B$, $E = E_A \bigcup E_B$, in which $V_A = \{x_1, x_2, \ldots, x_N\} := \{v_1, v_2, \ldots, v_N\}$ is the set of N state vertices, and $V_B = \{u_1, u_2, \ldots, u_M\} := \{v_{N+1}, v_{N+2}, \ldots, v_{N+M}\}$ is the set of M inputs, called input vertices; $E_A = \{(x_j, x_i)|a_{ij} \neq 0\}$ is the edge set between state vertices, and $E_B = \{(u_j, x_i)|b_{ij} \neq 0\}$ is the edge set between input vertices and state vertices.

According to Kalman controllability rank condition from control theory, the system (Eq. (1)) is controllable if and only if $rank(C = B, AB, \ldots, A^{N-1}B) = N$, where C is the controllability matrix. When it refers to the structural controllability, the A and B are often considered to be structured matrices and the system is structurally controllable if it is possible to choose the non-zero weights in A and B such that the system satisfies $rank(C) = N$.

3. Main results

The dynamical graph-based models became popular because of their potential application to network systems. A significant amount of work has been recently devoted to the controllability of such graph based models, e.g. for complex networks [12,11,10,19,24–26,13,27] and for multi-agent systems [28]. At first we give some concepts related to graph.

A path of $G(A, B)$ from a vertex v_{r_1} to a vertex v_{r_q} is a sequence of edges $(v_{r_1}, v_{r_2}), (v_{r_2}, v_{r_3}), \ldots, (v_{r_{q-1}}, v_{r_q})$ and the path is called as simple path if every vertex on the path occurs only once. A simple path is called a *U-root* path if the path has its begin vertex in $V_B = \{u_1, u_2, \ldots, u_M\} := \{v_{N+1}, v_{N+2}, \ldots, v_{N+M}\}$. A simple path from v_{r_1} to v_{r_q} comprising an additional

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