Physica A 442 (2016) 268–275

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Feature of topological properties in an earthquake network

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HIGHLIGHTS

- A seismic network is studied by considering the volume resolution and the temporal causality.
- The feature of the topological properties is mainly treated via various statistical quantities in the seismic networks in Japan.
- The topological properties improve by implementing the method and its technique from the seismic networks.

ARTICLE INFO

Article history:
Received 12 January 2015
Received in revised form 20 May 2015
Available online 15 September 2015

Keywords:
Earthquake network
Average clustering coefficient
Small-worldness
Global efficiency
Modularity
Assortative coefficient

ABSTRACT

Earthquake network is investigated by considering the volume resolution and the temporal causality. We perform the numerical computations for network metrics from seismic time series data taken in Japan. For our case, we mainly treat the feature of the topological properties via various statistical quantities such as the characteristic path length, the average clustering coefficient, the small-worldness, the cost efficiency, the global efficiency, the modularity, and the assortativity. The universal and irregular properties of statistical quantities in earthquake network do not find unambiguously, but it may be inferred that these topological properties improve by implementing the method and its technique from registered data of earthquake networks.

1. Introduction

Complex system has recently been applied to new research methods and high techniques [1–4] in various scientific fields such as the intermittent nature of turbulence [5], the various financial time series [6], the wavelet transform approaches [7], the growing and non-growing networks [8], and the earthquake phenomena [9], and so on. For the last two decades, the remarkable potential of complex networks to simulate and analyze the dynamical behavior of complex systems has gradually been an increasing trend in new fields of research in the social, natural, engineering, and medical sciences. The seismic phenomena in the network systems have performed diverse functions and also provided structural basis in the crust of earth. Characteristically, the network metrics of the seismicity may be supported to understand possible relevance of various seismic phenomena. The universal and irregular properties of statistical quantities in earthquake network do not find unambiguously as yet, and this is an open problem that comes to a settlement.

In network theory, the small-world and scale-free network models [10] have been studied widely in various applications of the scientific fields. The two network models have played a crucial role in complex phenomena. Of the many systems of

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http://dx.doi.org/10.1016/j.physa.2015.08.062
0378-4371/

current interest, the degree distribution for scale-free network is interesting because it follows the power law, while the random network has a Poisson distribution in the limit of large nodes. In a biological network, recent published papers have claimed that the protein–protein interaction networks have the characteristics of a scale-free network [11,12]. In scale-free networks, most proteins can deliberate only on a few interactions, while a small set of hubs relate to dozens of interactions.

Seismicity is a phenomenon of the dynamical behaviors in complex seismic time series [13,14], similar to a tsunami wave train. The shallow earthquake is well known to reorganize and analyze the distribution in the relevant area that leads to many aftershocks [15–18]. The Gutenberg–Richter law and the Omori law [19] have used to measure the number of aftershocks and analyzed the computational simulation of earthquakes by a theoretical formula. Particularly, Gutenberg and Richter found that the slip-size of faults or the seismic moment obeys a power law, while Omori found that the frequency of aftershocks decays in a power law. Furthermore, network theory is a topologically and dynamically useful tool for investigating and analyzing a earthquake system, which can be simplified as processes for storing and transmitting energy via the crust.

Several authors [20–22] have tested the validity of earthquake model by simulating specific configurations that are analytic methods to complex networks. That is, Abe and Suzuki have discussed and analyzed the method, which uses the concept of small-world and scale-free networks for seismic complexity. Abe and Suzuki also introduced a complex-network approach [20] to the earthquake, and they showed that the earthquake network behaves like a complex network. Peixoto and Prado [23,24] applied the complex network approach to a self-organized criticality model. They showed that the conservative regime exhibits a Poisson-like degree statistics, while the nonconservative has a broad power-law-like distribution of degrees, which reproduces the observed behavior of real earthquakes. Recently, Baek et al. [22] studied the earthquake network by considering the cell resolution and the temporal causality based on earthquake activity data for the Korean Peninsula [22]. They mainly estimated and analyzed several global network metrics. The detection of statistical quantities within earthquake networks is an open subject of great interest for the unknown dynamics governing seismicity as yet.

In this work, we study the topological robustness of the earthquake network against the spatial shift and the scale in a cubic cell which is located on a tectonic plate without boundaries. We mainly estimate the global network metrics such as the probability distribution of degree, characteristic path length, average clustering coefficient, small-worldness, cost efficiency, global efficiency, modularity, and assortativity, from the seismic data of Japan. Section 2 describes the theoretical method of complex networks. We treat the numerical calculation and its result in Section 3. Our result is summarized in Section 4.

2. Methodology

In this section, we mainly consider the theoretical background of the various global network metrics. First of all, in order to construct an earthquake network, we consider a network by segmenting the whole region into three-dimensional cells (cubes) and making a link between consecutive events. Each cell is regarded as node of a network, and the network constructed in that manner is basically directed, but we transform it into an undirected one because we focus on the topology of the network. The procedure is as follows: (i) we segment the whole region into cubic cells, each of which has the same size. (ii) Link two earthquakes occurring consecutively. (iii) If two consecutive events belong to the same cell, their link is disregarded. (iv) If two directed links form between two cells, the number of links is counted as one [9]. (v) By considering each cell as a node, we regard the links made by all events belonging to the cell with others in another cell as links of a network. First of all, since the degree k_i of a node i is the number of degrees to which it is directly connected, the average degree $\langle k\rangle$ of a network with N nodes is defined by $\langle k\rangle = N^{-1}\sum_{i=1}^{N} k_i$. First of all, we introduce the characteristic path length L given as

$$L = \frac{2}{N(N-1)}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} L_{ij}, \tag{1}$$

where L_{ij} is the shortest path length between nodes i and j. We consider that the diameter of the network is the largest of all the shortest path lengths. If the averaged shortest path length is proportional to $\log N$, it can be ascertained that the property of a network is satisfied the small-worldness.

The average clustering coefficient C is calculated as

$$C = \frac{1}{N}\sum_{i=1}^{N} C_i. \tag{2}$$

Here, the clustering coefficient C_i for a node i is defined as the fraction of links that exist among its nearest neighbor nodes to the maximum number of possible links among them. The global clustering coefficient C_g is defined as the transitivity ratio that is the fraction of the closed triangles over the whole triangles. A network is a small-world network if it has a large value of the average clustering coefficient and if its average shortest path length scales logarithmically with N [14]. Furthermore, a network has a small average clustering coefficient and having degree distribution of a power-law form is known as the scale-free network [25].

Using Eqs. (1) and (2), the small-worldness S_w can be computed as

$$S_w = \frac{C/C_{ran}}{L/L_{ran}}, \tag{3}$$