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# Interdependency enriches the spatial reciprocity in prisoner's dilemma game on weighted networks

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## HIGHLIGHTS

- An interdependent network-based game model with individual diversity is proposed.
- The interdependency is implemented by the utility coupling between partners.
- The diversity is characterized by the individual weighting on one lattice.
- The cooperation will be continuously elevated for weighted lattices.
- The nontrivial evolution of cooperation will be presented on the traditional lattice.

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## ABSTRACT

To model the evolution of cooperation under the realistic scenarios, we propose an interdependent network-based game model which simultaneously considers the difference of individual roles in the spatial prisoner's dilemma game. In our model, the system is composed of two lattices on which an agent designated as a cooperator or defector will be allocated, meanwhile each agent will be endowed as a specific weight taking from three typical distributions on one lattice (i.e., weighted lattice), and set to be 1.0 on the other one (i.e., un-weighted or standard lattice). In addition, the interdependency will be built through the utility coupling between point-to-point partners. Extensive simulations indicate that the cooperation will be continuously elevated for the weighted lattice as the utility coupling strength ($\alpha$) increases; while the cooperation will take on a nontrivial evolution on the standard lattice as $\alpha$ varies, and will be still greatly promoted when compared to the case of $\alpha = 0$. At the same time, the full $T - K$ phase diagrams are also explored to illustrate the evolutionary behaviors, and it is powerfully shown that the interdependency drives the defectors to survive within the narrower range, but individual weighting of utility will further broaden the coexistence space of cooperators and defectors, which renders the nontrivial evolution of cooperation in our model. Altogether, the current consequences about the evolution of cooperation will be helpful for us to provide the insights into the prevalent cooperation phenomenon within many real-world systems.



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## 1. Introduction

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Although the individual selfishness can help to reap the huger fruits once or twice during the competition and evolution, the collective cooperation may drive the population to reach the consensus and harvest the greatest gains, which may be contradictory to the Darwinian theory [1,2]. Thus, exploring the potential methods and means to foster the cooperation has become a pressing challenge, which also attracts the extensive interest and attention from inter-disciplinary researchers [3,4]. So far, quite a few mechanisms, such as kin selection [5], direct or indirect selection [6–8], group selection [9], spatial and network reciprocity and so on [10,11], have been devised to expound the collective social cooperation that originates from the interactions among individuals. In particular, Nowak and May presented a spatial game model which surprisingly enhanced the cooperation between selfish agents, and afterwards, spatial reciprocity provided a novel avenue to enrich the emergency of cooperation and trigged a large number of related works [12–15], for an instance, the reward or punishment [16–19], individual weighting [20–23], integration of environmental information [24–27], reputation [28,29], conformity and directional learning [30,31], neighborhood or topological setups [32–35] and community structures [36–39] are also added into the spatial games to further the emergency of cooperation.

Recently, Santos et al. [40,41] found that complex networks, especially for scale-free networks, offer a powerful framework to facilitate the evolution of cooperation, which greatly contributes to understand the persuasive cooperation behaviors. Accordingly, the network reciprocity becomes a potent tool to characterize the evolution among real-world systems [42–44]. As a further step, many real-world man-made or engineering systems are not isolated, but interconnected or interdependent, and the network model with interdependency and multiplexing has a better characterization for these systems [45–47]. Thus, exploring the impact of interdependency on the cooperation phenomenon has become an active topic [48]. Taking some examples, Wang et al. proposed a novel utility coupling mechanism to explain the role of interdependency in the evolution of cooperation [49]; Gómez-Gardeñes et al. utilized the superposition of payoffs to investigate the influence of multiplex networks [50]; Jiang and Perc discussed the role of interconnection between two-layered networks, in which one layer is a lattice and the other is a lattice, random network or scale-free network [51]; Szolnoki and Perc presented a kind of information sharing between two lattices to determine the strategy update probability [52]; two kinds of games are performed on different networks [53] in which strategy can be spread inside the same network with the probability $p$ or spread on the other one with $(1-p)$; Other related works are also found elsewhere [54–60], to name but a few.

However, in the above-mentioned works, players are often treated as identical peers in which individual heterogeneity is disregard. Nevertheless, the individual difference or diversity, such as age, position, reputation and so on, usually exists within the population [61]. How to represent this kind of difference among individuals is also an interesting problem [62]. In the spatial prisoner's dilemma game (PDG), Xia et al. endowed a specific weight, taking from three different kinds of probability distributions, to each individual so that their differentiation can be considered, and found that weighted lattices will further enrich the promotion of cooperation when compared to the standard lattice [63]. In this work, we will take use of two-layered lattices to characterize the interdependency between systems, where players on one lattice will be set to weighted and their correspondents on the other lattice will be un-weighted. Based on the prisoner's dilemma game and Fermi update rule, we have probed into the impact of different weight distribution on the cooperative behaviors under the interdependent networks. The results indicate that the cooperation will be greatly promoted, and the higher the individual difference, the larger the level of cooperation elevation.

The remainder of the paper is organized as follows. Firstly, Section 2 presents the networked game model considering the interdependency between two lattices, where players are weighted on one lattice but are not weighted on the other one. Secondly, Section 3 illustrates a large plethora of numerical results which persuasively demonstrate the promotion of cooperation, compared to the traditional spatial reciprocity. Finally, some concluding remarks are drawn in Section 4.

## 2. Weighted PDG model on interdependent lattices

We use a two-layered $L \times L$ lattice setup shown in Fig. 1 to characterize the interdependency between networks, players are arranged on intersections (i.e., nodes) of lattices and here each node can only contain one player. Without loss of generality, it is assumed that the lower layer is a standard lattice on which each node is considered as an identical peer, while the upper layer is a weighted lattice where each node is furnished with a different weight so as to characterize the individual discrepancy in the position, reputation, age and so on. Particularly, we hypothesize that any player's weight $(\omega_x)$ is equal to $\omega_x = 1 + \xi_x$ where $\xi_x$ is a stochastic value taking from a specific distribution. Here, three different distributions, including uniform, exponential and power-law distribution, are adopted in the following section,

$$\xi_x = A * (-2\chi + 1) \quad (1)$$

$$\xi_x = A * (-\ln \chi - 1) \quad (2)$$

$$\xi_x = A * (\chi^{-0.5} - 2) \quad (3)$$

where $\chi$ is a uniformly distributed random number within the interval [0, 1], and $\int_{-1}^{1} \xi_x(\chi)\mathrm{d}\chi = 0$ in the above-mentioned case so that the average weight among the whole population is still kept to be 1.0, comparable to the standard case in which $\omega_x$ is equal to 1.0 for any player; The parameter $A$ $(0 \le A \le 1)$ is a tunable parameter representing the amplitude of

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