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First results on applying a non-linear effect formalism to alliances between political parties and buy and sell dynamics

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HIGHLIGHTS

- We use fermionic operators to describe an open model of alliances in politics.
- From a given Hamiltonian we derive a nonlinear system of equation.
- The same Hamiltonian is used to describe dynamics of buying and selling.

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ABSTRACT

We discuss a non linear extension of a model of alliances in politics, recently proposed by one of us. The model is constructed in terms of operators, describing the *interest* of three parties to form, or not, some political alliance with the other parties. The time evolution of what we call *the decision functions* is deduced by introducing a suitable Hamiltonian, which describes the main effects of the interactions of the parties amongst themselves and with their *environments*, which are generated by their electors and by people who still have no clear idea for which party to vote (or even if to vote). The Hamiltonian contains some non-linear effects, which takes into account the role of a party in the decision process of the other two parties.

Moreover, we show how the same Hamiltonian can also be used to construct a formal structure which can describe the dynamics of buying and selling financial assets (without however implying a specific price setting mechanism).

1. Introduction

In a recent paper, [1], a model of interaction between political parties has been proposed. The model describes a decision making procedure, deducing the time evolution of three so-called *decision functions* (DFs), one for each party considered in our system. These functions describe the interest of each party to form or not an alliance with some other party. Their decisions are driven by the interaction of each party with the other parties, with their own electors, and with a set of undecided voters (i.e. people who have not yet decided to vote for which party (if at all they decide to vote)). The approach

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adopted in Ref. [1] uses an operatorial framework (see also Ref. [2]), in which the DFs are suitable mean values of certain number operators associated to the parties. The dynamics are driven by a suitable Hamiltonian which implements the various interactions between the different actors of the system.

The limitation of the model, as described in Ref. [1], is that the Hamiltonian is quadratic and, as a consequence, the equations of motion are linear. This simplifies quite a bit the analysis of the time evolution of the system. In fact an exact solution can be deduced in that case, but the price we pay is that the model is not entirely realistic, since the Hamiltonian does not include contributions which might be relevant in a concrete situation. In this paper we introduce several *non-linear contributions* in the model, and we solve, adopting a suitable approximation, the related non-linear differential equations. These non-linear terms are needed to introduce in the model some sort of three-body interactions, which were not included in Ref. [1]. The reason why these terms are interesting is because they describe (please see below for more details) the role of, say, the first party ($\mathcal{P}_1$), in the explicit strength of the interaction between the other two parties, $\mathcal{P}_2$ and $\mathcal{P}_3$. This is important, since it is natural to assume that the DFs of both $\mathcal{P}_2$ and $\mathcal{P}_3$ also depend on what $\mathcal{P}_1$ is doing.

It is important to notice that not many contributions exist in the mathematical and physics literature on politics, and only very few of them adopt a quantum mechanical (or operator) point of view, as the one used in Ref. [1]. We refer to Refs. [3–6] for some recent and not so recent contributions on this topic.

After a long discussion on politics, we also show how the same Hamiltonian can be used, with just some minor changes, to deduce the dynamics of a buy-and-sell financial system.

The paper is organized as follows: In Section 2 we introduce the model, we derive the differential equations and we propose an approximation scheme to solve them. In Section 3 we show how to model a simple financial system using the same general settings. Section 4 contains our conclusions. To keep the paper self-contained, and to make it also more readable to those who are not familiar with quantum mechanics, we have added an Appendix where a few crucial aspects of operators and quantum dynamics are reviewed.

2. Modeling alliances in politics and its dynamics

In this section we discuss the details of our model and we will first construct the vectors describing the players and the Hamiltonian of the system. We then deduce the differential equations of motion. To keep the paper self contained, we recall first a few important facts which were already discussed in Ref. [1].

In our system we have three parties, $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$, which, together, form the system $\mathcal{S}_{\mathcal{P}}$. Each party has to make a choice, and it can choose only 'one' or 'zero', which corresponds respectively to either *form a coalition* or not. This is, in fact, the only aspect of the parties we are interested in. Hence, we have eight different possibilities, to which we associate eight different and mutually orthogonal vectors in a n eight-dimensional Hilbert space $\mathcal{H}_{\mathcal{P}}$. These vectors are $\varphi_{i,k,l}$, with $i, k, l = 0, 1$. As an example, the first vector, $\varphi_{0,0,0}$, describes the fact that, at $t = 0$, no party wants to ally with the other parties. Of course, this attitude can change during the time evolution. What is interesting to know is: how does this attitude change? And how can one describe this change? Let us consider another example. For instance, $\varphi_{0,1,0}$, describes the fact that, at $t = 0$, $\mathcal{P}_1$ and $\mathcal{P}_3$ do not want to form any coalition, while $\mathcal{P}_2$ does. $\mathcal{F}_\varphi = \{\varphi_{i,k,l},\ i, k, l = 0, 1\}$ is an orthonormal basis for $\mathcal{H}_{\mathcal{P}}$. A generic vector of $\mathcal{S}_{\mathcal{P}}$, for $t = 0$, is a linear combination of the form

$$\Psi = \sum_{i,k,l=0}^{1} \alpha_{i,k,l}\varphi_{i,k,l}, \tag{2.1}$$

where we assume $\sum_{i,k,l=0}^{1} |\alpha_{i,k,l}|^2 = 1$ in order to normalize the total probability, [7]. In particular, for instance, $|\alpha_{0,0,0}|^2$ represents the probability that $\mathcal{S}_{\mathcal{P}}$ is, at $t = 0$, in a state $\varphi_{0,0,0}$, i.e. that $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ have chosen '0' (no coalition).

As in Ref. [1], and for the same reasons (see below), we construct the vectors $\varphi_{i,k,l}$ in a very special way, starting with the vacuum of three fermionic operators, p_1, p_2 and p_3, i.e. three operators which, together with their adjoint, satisfy the canonical anticommutation relation (CAR) $\{p_k, p_l^\dagger\} = \delta_{k,l}$ and $\{p_k, p_l\} = 0$. Here $\{x, y\} = xy + yx$, for all pairs x and y. More in detail, $\varphi_{0,0,0}$ is such that $p_j\varphi_{0,0,0} = 0, j = 1, 2, 3$. The other vectors $\varphi_{i,j,k}$ can be constructed acting on $\varphi_{0,0,0}$ with the operators $p_1^\dagger$, $p_2^\dagger$ and $p_3^\dagger$:

$$\varphi_{1,0,0} = p_1^\dagger\varphi_{0,0,0}, \qquad \varphi_{0,1,0} = p_2^\dagger\varphi_{0,0,0}, \qquad \varphi_{1,1,0} = p_1^\dagger p_2^\dagger\varphi_{0,0,0}, \qquad \varphi_{1,1,1} = p_1^\dagger p_2^\dagger p_3^\dagger\varphi_{0,0,0},$$

and so on. Let now $\hat{P}_j = p_j^\dagger p_j$ be the so-called *number operator* of the jth party, which is constructed using p_j and its adjoint, $p_j^\dagger$. Since $\hat{P}_j\varphi_{n_1,n_2,n_3} = n_j\varphi_{n_1,n_2,n_3}$, for $j = 1, 2, 3$, it is clear that φ_{n_1,n_2,n_3} are eigenvectors of these operators, while their eigenvalues, zero and one, correspond to the only possible choices admitted for the three parties at $t = 0$. This is, in fact, the main reason why we have used here the fermionic operators p_j: they automatically produce only these eigenvalues. Our first effort now consists in *giving a dynamics* to the number operators $\hat{P}_j$, following the general scheme proposed in Ref. [2]. Hence, we look for an Hamiltonian H which describes the interactions between the various constituents of the system. Once H is given, we can compute first the time evolution of the number operators as $\hat{P}_j(t) := e^{iHt}\hat{P}_j e^{-iHt}$, and we can then ascertain their mean values on some suitable state describing the system at $t = 0$, in order to get what we have already called *decision functions*, (DFs) (please see below). The *rules* needed to write down H are described in Ref. [2], and adopted in Ref. [1] where it is also discussed why the three parties are just part of a larger system which must also include the set of electors. In fact, it