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A superstatistical model of vehicular traffic flow

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HIGHLIGHTS

- It is shown that superstatistics may be an important tool in traffic flow analysis.
- Short and long time scales are computed for the vehicular flow time series data.
- The q value computed from q-Gaussians characterizes the highway segment.
- The q value obtained from the beta distribution defines a given time series data.
- Traffic flow may be characterized by long-range interactions.

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ABSTRACT

In the analysis of vehicular traffic flow, a myriad of techniques have been implemented. In this study, superstatistics is used in modeling the traffic flow on a highway segment. Traffic variables such as vehicular speeds, volume, and headway were collected for three days. For the superstatistical approach, at least two distinct time scales must exist, so that a superposition of nonequilibrium systems assumption could hold. When the slow dynamics of the vehicle speeds exhibit a Gaussian distribution in between the fluctuations of the system at large, one speaks of a relaxation to a local equilibrium. These Gaussian distributions are found with corresponding standard deviations $1/\sqrt{\beta}$. This translates into a series of fluctuating beta values, hence the statistics of statistics, superstatistics. The traffic flow model has generated an inverse temperature parameter (beta) distribution as well as the speed distribution. This beta distribution has shown that the fluctuations in beta are distributed with respect to a chi-square distribution. It must be mentioned that two distinct Tsallis q values are specified: one is time-dependent and the other is independent. A ramification of these q values is that the highway segment and the traffic flow generate separate characteristics. This highway segment in question is not only nonadditive in nature, but a nonequilibrium driven system, with frequent relaxations to a Gaussian.

1. Introduction

Real life dynamics of a certain system, e.g. vehicular traffic flow, is often an interwoven dynamics of multiple of those superimposed on one another. If the system demonstrates driven nonequilibrium system characteristics, with multiple time scale separation, then one may suspect of the presence of superstatistics. Currently, a well-separated two or more time scales is an indication of the theory. The superstatistics theory suggests that there should be a fast time scale where there are

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many equilibrium states whose dynamics may very well be described by the Boltzmann–Gibbs statistical mechanics, whose parameter distributions could be described by Gaussians. Then, there is another time scale, a much slower one, whose slow dynamics is accounted for by the environmental changes, for example. To go along with the theory, this change is caused by a parameter, known as the local inverse temperature, β. This inverse temperature β could be any parameter that is representative of the local variance, found by windowing the whole time series data. Since this β is a statistical parameter out of local windows, and whose long term change could also be defined by another global statistics, the term statistics of the statistics, or superstatistics would fit well.

If one overviews the traffic flow in a thermodynamic context, even before delving into the theory of superstatistics, Ref. [1] is a good starting point. Here the authors mention a continuous particle system that is under psychological forces, or interactions. All the interactions are assumed to be of short-range, hereby shearing much of the connection with the Tsallis thermostatistics [2,3], over a closed-loop ring. The two-body repulsion potential is given as a function of the vehicle locations around this ring, and a Hamiltonian of speeds and locations of the particles is provided. The kinetic energy of the Hamiltonian is specified with respect to an optimal speed. An inverse temperature parameter is named mental strain coefficient, and taken constant for a given part of a freeway. This study claims that the steady-state properties are in good agreement with the relevant properties of traffic flow. No long-range interactions are considered. Three different analyses are performed for three distinct (constant) pseudo-temperature parameters, or mental strain coefficients, generating specific distance clearance distributions for vehicular congestions.

Ref. [4] deals with the traffic flow, in a similar vein, determining interaction potentials through steady-state statistics. Only that this time real-life data of Dutch two-lane freeway A9 is used. Pseudo-temperature values are computed and tabulated. But all of these parameters reflect the specific cases, and not a fluctuating β as in superstatistical theory.

Another study [5] has the vehicles distributed not over a closed-loop band but over the surface of a sphere, called a unit sphere. The novelty is that the time spectral rigidity is found via Random Matrix Theory, since it provides a general overlook into the eigenvalues of the random matrix ensembles. This time, headway distributions are obtained for specific inverse temperatures. And the study [6] follows a scenario similar to Refs. [1,4], only that inter-vehicle gap statistics is analyzed at signal-controlled crossroads.

The traffic flow has also been modeled through thermostatistical models as well as traditional techniques that are not mentioned in this paper. A typical unconventional approach is by Ref. [7]. Here, the driven systems in a thermodynamic framework are being discussed. Particles are identified as vehicles. The traffic flow is described by Bando's optimal velocity model in terms of forces. Bando's optimal velocity model allows an internal energy of the car system in a thermodynamic framework. The size of the car cluster is assumed to be a stochastic variable. Hence the solutions range from a fixed-point solution to a free-flow phase in a limit cycle. A detailed analysis on distinguishing the characteristics of a traffic flow is provided, where the system is specified as either driven or equilibrium systems.

As for the superstatistics concept, it is a relatively new idea, whose origins and the theory may be found in Refs. [8–10]. A more comprehensive discussion is found in Ref. [11]. Some of the key points were duly explained. Some of these key points are the fingerprints of the superstatistical dynamics, the ratio of the time scales, and the distribution of the inverse temperature β. The case study is a turbulent Taylor–Couette flow, where the longitudinal velocity differences are processed. One practical finding for the log-normal distribution of the parameter β is that the Tsallis q variable is linked to the flatness of this distribution.

Another fundamental work in the theory of superstatistics is by Ref. [12]. This work sets the standards for superstatistical analysis of any time series. Inspired by the theory of statistical hypothesis testing, the method proposed in the manuscript computes the parameters of a data set. But most importantly, the proposed method checks for the validity of the superstatistical model assumptions. The practical outcome of this cornerstone study is that, for any superstatistical assumption to be valid, two conditions must be satisfied. The first condition states that the ratio of the slow time scale to the fast one should tend to zero. The latter condition is on the deviation of the Gaussianity, designated by ε, and dictates that it too should tend to zero.

A recent study [13] on share price returns has focused on determining a proper superstatistical model on a range of time scales. By allowing the time scale to move from minutes to a larger scale of days, a transition from lognormal to chi-square superstatistics has been observed. Even a general model was proposed over a control parameter κ, interpolating a hybrid superstatistical structure. There is a mention that, by varying the system parameters, a range of q values may be obtained. The factors such as time scale of returns, intraday periodicity, and the sector of shares all affect this range of q values.

There are numerous applications of superstatistical theory on data analyses, such as sea-level fluctuations [14], high energy physics [15], modeling train delays [16], environmental atmospheric turbulence [17], temperature fluctuations [18], metastasis and cancer survival [19], currents in complex polymers [20], and headway distributions [21].

Of the superstatistical and the traffic flow literature, there are two research papers that are the most relevant: [21] and [22]. Ref. [22] studies the clearance distributions of the traffic flow and the time-headway distributions. Using an analogy, traffic states are considered to be phase changes of matter, as a function of vehicle density. An analytic expression is derived for the spacing distributions that interpolate from the Poisson distribution. Here, the fast dynamics is represented by the vehicle velocity and the slow dynamics by the traffic density. It is stated that any statistic of a congested traffic may be expressed as an average of another statistic for a Gaussian random ensemble over the local mean level spacing.

In this paper, we first investigate the probability distributions of the speed variable at a local point selected from a highway segment. If the space is Euclidean, the memory is Markovian and the time is continuous; the distribution of a given