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Anomalous Brownian refrigerator

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HIGHLIGHTS

- We study the stochastic thermodynamics of Brownian refrigerator.
- Apart from refrigerator, the system can work as heaters and heat engine, depending on ratio of the bath temperatures and cycle time.
- Distributions of stochastic COP and efficiency are obtained using numerical simulation.
- Stochastic efficiency/COP distributions are shown to have power law tails and they are dominated by fluctuations.

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ABSTRACT

We present a detailed study of a Brownian particle driven by Carnot-type refrigerating protocol operating between two thermal baths. Both the underdamped as well as the overdamped limits are investigated. The particle is in a harmonic potential with time-periodic strength that drives the system cyclically between the baths. Each cycle consists of two isothermal steps at different temperatures and two adiabatic steps connecting them. Besides working as a stochastic refrigerator, it is shown analytically that in the quasistatic regime the system can also act as stochastic heater, depending on the bath temperatures. Interestingly, in non-quasistatic regime, our system can even work as a stochastic heat engine for certain range of cycle time and bath temperatures. We show that the operation of this engine is not reliable. The fluctuations of stochastic efficiency/coefficient of performance (COP) dominate their mean values. Their distributions show power law tails, however the exponents are not universal. Our study reveals that microscopic machines are not the microscopic equivalent of the macroscopic machines that we come across in our daily life. We find that there is no one to one correspondence between the performance of our system under engine protocol and its reverse.

1. Introduction

Thermodynamics of micro- and nano-scale systems exhibits distinctly different features from that of large systems due to influence of large thermal fluctuations [1]. Typical energy changes are of the order of thermal energy per degree of freedom and consequently thermodynamics has to be modified at micro-scale. These systems can be theoretically analyzed using stochastic thermodynamics. Exchange of energy between the particle and its surroundings becomes stochastic and yet one can clearly formulate the notion of work, heat and entropy production for a given microscopic trajectory of the particle [2–9]. Recently obtained exact results (fluctuation theorems [11,10,12–15]) put constraints on the distributions

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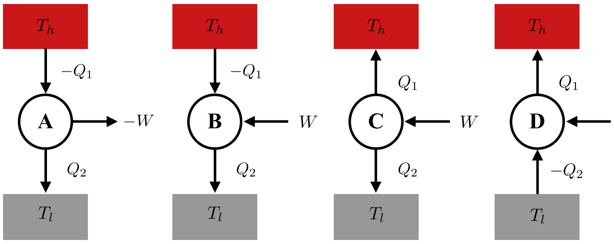

Fig. 1. Four thermodynamically possible machines working between a hot (with temperature T_h) and a cold (with temperature T_l) thermal baths: A: Takes heat Q_1 from hot bath and converts it partially into work W and supplies the rest in the form of heat Q_2 to the cold bath (heat engine). B: Takes heat Q_1 from the hot bath and with the help of work W on it, supplies heat Q_2 to the cold bath. C: Converts work W on it, to heats Q_1 and Q_2 that enter into the hot and cold baths respectively. (B) and (C) are called heater of type-II (or, heater-II) and heater of type-I (or, heater-I) respectively. D: Takes heat Q_2 from the cold bath and with the help of work W on it, supplies heat Q_1 to the hot bath (refrigerator). When work is being done on (by) the system, it is positive (negative). When the system releases heat it is positive and negative otherwise. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

of the above mentioned stochastic quantities and are valid for systems driven far from equilibrium. These theorems transform thermodynamic inequalities into equalities. This area has become even more interesting with the development of experimental techniques. Using single-colloidal particle experiments several new key concepts have been verified. Information to energy conversion and validation of generalized Jarzynski equality [16], Landauer erasure principle [17], universal features in the energetics of symmetry breaking [18] are to name a few. Micron sized heat engines have been experimentally realized by optically controlled motion of trapped colloidal particle [19,20].

There are several extensive studies on single bath nano-machines e.g., information machines (that can produce work using available information) [21,22] and molecular motors/thermal ratchets [23,24]. Molecular motors are omnipresent in cellular as well as tissue level of many living organisms. They are efficient enough to extract energy from a highly fluctuating environment and to convert it into mechanical work for cellular and/or intra-cellular logistics [25].

The conversion of energy into mechanical work repeatedly along a cycle working within multiple thermal bath is almost everywhere in our day-to-day life, spanning huge range of length and time scales. For example, starting from high pressure steam locomotives, to a *drinking bird* [26] and even biochemical reaction pathways for cellular respiration mechanism [27,28] producing useful energy from nutrients—everywhere energy is transformed into mechanical work. Similarly, by reversing the cycle, we see that in various processes mechanical work is used to transfer heat from a cold source to a warm sink with an objective to cool down the cold source further (refrigerator) or to heat up the warm sink warmer (heat pump). Though it is very important to study the work-energy (or, vice versa) conversion in all relevant scales, due to lack of experimental techniques for micro or nano world, it is relatively well explored in macro scale.

Heat engines and refrigerators at nano-scale are a subject of current study [29–37]. Detailed theoretical treatment of *Carnot-type* micro heat engine, involving both quasistatic and non-quasistatic (i.e., finite cycle time) features, have also been documented [33–38]. These features reveal the fundamental differences between micro and macro heat engines due to thermal fluctuations, reflected in the distributions of various thermodynamic quantities (e.g., work, heat exchange, efficiency, etc.). Unlike macro heat engines, it has been shown that the system can work as a heat engine if the ratio of hot and cold bath temperatures is larger than a critical value [38]. Moreover, in non-quasistatic regime, the system works as a heat engine for cycle times larger than a critical value. Both the thresholds depend on the system parameters. Fluctuations in thermodynamic quantities including efficiency of the system calculated over a large number of trajectories are significant not only in non-quasistatic regime but also in quasistatic regime, which is in clear contrast to the macro engines. Several trajectories violate typical expectations from the second law of thermodynamics [39,40]. The non self-averaging nature of fluctuations in stochastic efficiency and other quantities requires detailed understanding of full probability distributions as opposed to the average behavior [41,42]. Large deviation properties of such distributions are recently under theoretical investigations [45–51]. Research on fluctuation relations for heat engines [52–54] are being pursued. It may also be noted that, for some of the heat engines studied so far, one may or may not recover Carnot result in the quasistatic regime. However, fluctuation theorems provide a bound on efficiency of an engine valid for any finite time cycle. Recently, novel theoretical approaches to capture the statistical properties of stochastic efficiency of micro heat engines and mesoscopic thermoelectric engines with broken time-reversal symmetry are being developed, particularly at long time limit [43,44], claiming universal properties of the large deviation function related to the statistics of stochastic efficiency. To our knowledge, so far, no such study exists for micro-refrigerators.

In this article we will focus on *Carnot-type* single-particle refrigerator and its stochastic features. The refrigeration protocol used here is similar to the micro heat engine protocol used in Ref. [38] but is running backward in time. We believe that our model system is experimentally realizable using the technique already being used for micro heat engines. We find new insights into far from equilibrium features of the concerned system. For example, a major outcome of the present study is the variety of different modes of operation for such systems under the protocol. In Fig. 1 we describe all the modes of operations which are thermodynamically possible for a system that works cyclically between a hot and a cold heat bath. Other four possibilities for heat exchanges and work are ruled out due to violation of First and Second laws of thermodynamics.