



A comparison of LMC and SDL complexity measures on binomial distributions



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HIGHLIGHTS

- SDL and LMC complexity measures are applied to repeated trials binomial distribution.
- Increasing the number of trials, the informational entropy decreases.
- Maximum SDL and LMC complexity measures occur for unbalanced success probability of the trials.
- SDL and LMC complexity measures decrease with the number of trials.
- The maximum values of SDL and LMC measures do not depend on the number of trials.

ARTICLE INFO

Article history:

Received 15 June 2015

Received in revised form 4 October 2015

Available online 23 October 2015

Keywords:

Binomial
Complexity
Information
Measure
Probability

ABSTRACT

The concept of complexity has been widely discussed in the last forty years, with a lot of thinking contributions coming from all areas of the human knowledge, including Philosophy, Linguistics, History, Biology, Physics, Chemistry and many others, with mathematicians trying to give a rigorous view of it. In this sense, thermodynamics meets information theory and, by using the entropy definition, López-Ruiz, Mancini and Calbet proposed a definition for complexity that is referred as LMC measure. Shiner, Davison and Landsberg, by slightly changing the LMC definition, proposed the SDL measure and the both, LMC and SDL, are satisfactory to measure complexity for a lot of problems. Here, SDL and LMC measures are applied to the case of a binomial probability distribution, trying to clarify how the length of the data set implies complexity and how the success probability of the repeated trials determines how complex the whole set is.

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1. Introduction

Life emerged from complex interactions between pieces of matter, generating dynamical processes that need more than reductionist science to be explained [1,2]. In spite of this, the reductionistic approach gave a strong development to all natural sciences along the former two centuries.

However, during the second half of the 20th century the onset of a new science unification movement started when the study of dissipative structures in chemical reactions [3] generated the ideas of “Deterministic chaos” [4], “Self-organization” [5] and “Self-organizing criticality” [6] that, together with the development of Nonlinear Dynamics [4], supported by computational methods, became popular.

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Combined with the Edgard Morin ideas about “Complex thinking” [7], these new ways of thinking pervaded several areas of the human knowledge, decisively contributing to model important problems related to the onset of surprising or catastrophic events as traffic jams, crowd behavior, tsunamis and earthquakes [8].

In this paper, by using a simple probability distribution, it is shown that the size of a system or a set of data is not the origin of complex behavior. Uncertainty and interaction, as exhaustively discussed in the literature, combined with nonlinearities, are the main agents of complexity [9].

In the next section, the concepts of informational entropy, disorder, order, SDL complexity and LMC complexity [10,11] are presented. Their applications to binomial distributions are explained followed by a section with the numerical results related to number of trials and success probability at each trial. A conclusion section closes the work.

2. Binomial distribution: entropy, SDL and LMC complexity measures

To start the calculations, N binomial independent trials with success probability p are considered. Consequently, the probability density function is discrete and given by:

$$p(i) = \binom{N}{i} p^i (1-p)^{N-i}, \quad (1)$$

for $i = 0, 1, \dots, N$ [12].

With the probability function in hand, it is possible to calculate the informational entropy, E , in bits per trial, by using:

$$E = - \sum_{i=0}^N p_i \log_2 p_i, \quad (2)$$

having its maximum value given by: $E_{\max} = \log_2 N$ [13].

Consequently, the disorder, Δ , measuring the thermodynamic equilibrium [10,11], is calculated by using:

$$\Delta = \frac{E}{E_{\max}}. \quad (3)$$

As stated in Ref. [10], combining disorder, Δ and order ($1 - \Delta$), SDL complexity measure, C_{SDL} , can be defined as:

$$C_{SDL} = \Delta(1 - \Delta), \quad (4)$$

meaning that maximum complexity corresponds to an equal balance between order and disorder.

LMC complexity measure contains a term called disequilibrium D that in SDL complexity measure was replaced by the order term ($1 - \Delta$). The disequilibrium D measures the distance between the probability distribution and the equiprobable one, and is defined by:

$$D = \sum_{i=0}^N (p_i - 1/N)^2. \quad (5)$$

Then, LMC complexity measure is given by:

$$C_{LMC} = \Delta D. \quad (6)$$

3. Numerical experiments

Based on the simple theoretical points related in the former section, some questions can be discussed. The first is about how the increase of repeated independent binomial trials changes informational entropy of the experiment.

For some different values of p , Fig. 1 shows how informational entropy, E , depends on the total number of trials, N .

Other point to be discussed is how the success probability of each trial, p , changes informational entropy, E , SDL, C_{SDL} , and LMC, C_{LMC} , measures considering the number of trials, N , as a parameter. Fig. 2 shows these functions considering the number of trials equal to 1, 5 and 10.

Observing Fig. 2, it seems to be interesting to observe how the parametrization by the number of trials changes SDL and LMC complexity measures.

Figs. 3 and 4 show the result of the numerical experiment of changing the number of trials, observing the behavior of SDL complexity (Fig. 3) and LMC complexity (Fig. 4).

4. Conclusions

In spite of using only classical works and a very simple example, this work provides several interesting facts about complexity. The first is a consequence of the observation of Fig. 1, i.e., as the number of independent trials increases, the informational entropy of the whole set of the results decreases, indicating that the computational complexity [14] decreases as the length of the data set increases.

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