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Critical exponents of a self-propelled particles system

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HIGHLIGHTS

- We model self-propelled particles system used to study the live organisms motion.
- Important parameters in this system are velocity, interaction radius and density.
- We estimate a set of critical exponents as a function of these parameters.
- We find that such parameters, in turn, influences the values of critical exponents.
- Critical exponents obtained satisfy the hyperscaling relationship.

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ABSTRACT

The Vicsek model of self-propelled particles is an important tool in the study of the collective motion of live organisms. The model consists of particles that move with a constant velocity and adopt, in a region called the zone of repulsion, the average motion direction of their neighbors disturbed by an external noise. A second-order phase transition from a disordered state, with motion in random directions, to an ordered motion state was observed. In this work, we have estimated, using finite-size scaling arguments, the critical exponents β, γ and ν of the original Vicsek model as a function of parameters important to the model, such as the orientation radius size, density, and velocity modulus. Our results show that the critical exponents depend greatly on these parameters.

1. Introduction

Collective behavior is a fascinating characteristic observed in biological systems [1]. Examples of organized animal groups observed in nature include flocks of birds [2], herds of mammals [3], and schools of fish [4]. In these aggregates, global collective behavior emerges as a result of the local interactions among individuals in close proximity to one another where each individual in the group engages in behavior as a function of the position and velocity of its nearest neighbors and, as a natural consequence, aligns itself and moves in a common direction. In this context, several studies have been employed in order to describe the main features of collective motion in such systems [5–12]. Self-propelled particle dynamics was employed by Vicsek et al. [5] to describe the collective phenomena of biological organisms. In their model, each particle moves with a constant velocity and at each time step adopts the average motion direction of its neighboring within an interaction radius r centered at the actual position of the particle. The particles that access this interaction zone will have their directions updated with some random perturbation added to their motion. When noise is introduced, the Vicsek model

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displays a phase transition, controlled by the noise intensity, a phase transition from a disordered state, with random motion directions, to an ordered state wherein the individuals move coherently in the same common direction. In their work, Vicsek et al. classify the nature of this transition as second-order, which was later confirmed by other authors [13,14].

Random perturbations in the motion direction in self-propelled models represent errors committed by the particles in evaluating the real motion direction of its neighbors. There are two types of noise—intrinsic and extrinsic, also referred to as angular and vectorial respectively. We can define intrinsic noise as arising from the errors committed when particles try to follow the average motion direction, already perfectly calculated, of neighboring particles. On the other hand, for the case of extrinsic or vectorial noise the errors arise from the evaluation of each particle–particle interaction; in this case, the noise is added directly into the interactions between the particle i and each one of its neighbors. The two different ways in which the noise is introduced produces different types of phase transitions. The orientational transition in the Vicsek model with intrinsic noise was shown to have a continuous character, whereas models with extrinsic noise have been shown to lead to a discontinuous phase transition.

Although the study of self-propelled particles systems is well established in the literature, we are not aware of a detailed study of critical exponents. Vicsek et al. [5] calculated the critical exponent associated with the order parameter and found $\beta = 0.45(7)$, for a given density $\rho = 0.4$ and a velocity $v_0 = 0.03$. They did not consider the exponents ν and γ, of correlation length and susceptibility respectively. In a more thorough analysis using finite-size scaling arguments, Baglietto and Albano [15] obtained, by collapsing data onto a single curve, the static exponents of Vicsek model for systems of different sizes and densities in the low-velocity regime. They found $\beta = 0.45(3)$, $\gamma = 2.3(4)$ and $\nu = 1.6(3)$. Important results were found in Refs. [14,16], where the coupled effect between initial velocity and density of particles has been shown to play an important role in changing the behavior of order parameter. In this same context, Tarras et al. [17] proposed a Vicsek-type model, where they added a repulsion zone in order to avoid collisions between the particles. They calculated the critical exponents β and δ for different repulsion radiuses and densities and showed that exponents depend greatly on the size of these parameters, indeed the model proposed by them indicate the non-universality of these exponents.

Important parameters in the original Vicsek model are: the velocity of particles, the interaction radius, and the density. High velocities can speed up the interactions; the gradual increase of the interaction radius increases the number of neighbors of each particle contributing to group cohesion; and large densities contribute to the appearance of ordered motion.

Our contribution in this paper is to estimate the static critical exponents of the original Vicsek model as a function of density, velocity and orientation radius. We employ finite-size scaling arguments in order to obtain a complete picture in terms of each of these parameters. We have demonstrated that the values of parameters ρ, r and v_0 influence significantly the values of the critical exponents, indicating that the critical exponents depend on the details of the model.

The rest of this paper is organized as follows: The detailed formulation of the Vicsek model [5] is presented in Section 2. In Section 3, the simulation parameters and a brief description of the finite-size scaling theory is discussed. We present the results of our simulations in Section 4 and the main conclusions are presented in Section 5.

2. The Vicsek model—a brief description

In a two-dimensional space, initially N self-propelled particles are characterized by their position vector $\mathbf{x}_i(t)$ in t time. Their directions are distributed randomly in a square lattice L, with periodic boundary conditions and a constant velocity ($v_0 = 0.03$). The interactions between particles occur, at each time step, in a region of radius $r = 1$, where each individual particle assumes the average motion direction of all its neighbors. The positions are updated by the equation,

$$\mathbf{x}_i(t+\tau) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\tau, \tag{1}$$

where $\tau = 1$ is the time interval. The direction of an individual particle i, with some uncertainty, is specified by the angle $\theta_i(t+\tau)$, which is updated by the following interaction rule,

$$\theta_i(t+\tau) = \langle \theta(t) \rangle_{(r=1)} + \xi(t) \times \eta \tag{2}$$

where $\langle \theta(t) \rangle_{(r=1)}$ denotes the average motion direction of the neighboring particles, ξ is a random variable uniformly distributed between $[-\pi, \pi]$, and η is the strength of noise.

The order parameter ψ is defined as the absolute value of the normalized average velocity, i.e.,

$$\psi = \frac{1}{N v_0} \left| \sum_{i=1}^{N} \mathbf{v}_i(t) \right|, \tag{3}$$

where ψ varies between [0, 1]. ψ is zero when the motion directions of the particles are random, in this case no average alignment of the velocities is verified. When ψ equals one, the particles tend to move towards the same direction. The shape of the curve of ψ (see Fig. 1) suggests, as pointed out in Ref. [5], the presence of a second-order phase transition in the system.