Physica A 444 (2016) 897–904

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Antiferromagnetic majority voter model on square and honeycomb lattices

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HIGHLIGHTS

- We proposed an antiferromagnetic version of the majority voter model on square and honeycomb lattice.
- We evaluate with great accuracy the critical point using combination of three cumulants.
- We found that the static critical exponents are consistent with values of the 2D Ising model.

ARTICLE INFO

Article history:
Received 15 September 2015
Received in revised form 20 October 2015
Available online 29 October 2015

Keywords:
Antiferromagnetic
Majority voter model
Finite-size scaling
Ising model
Universality class

ABSTRACT

An antiferromagnetic version of the well-known majority voter model on square and honeycomb lattices is proposed. Monte Carlo simulations give evidence for a continuous order–disorder phase transition in the stationary state in both cases. Precise estimates of the critical point are found from the combination of three cumulants, and our results are in good agreement with the reported values of the equivalent ferromagnetic systems. The critical exponents $1/\nu$, γ/ν and β/ν were found. Their values indicate that the stationary state of the antiferromagnetic majority voter model belongs to the Ising model universality class.

1. Introduction

The Majority voter model (MVM) is a simple non-equilibrium Ising-like system, proposed as a way to simulate opinion dynamics. The collective behaviour of the voters shares many aspects with the well-established theory of non-equilibrium phase transitions and results from simulations can be analysed similarly [1]. In the standard MVM, the system evolves following a dynamics where each "voter" (spin) assumes the same opinion as the majority of its neighbours, with probability $(1+x)/2$ and assumes the opposite opinion, with probability $(1-x)/2$. Here x is the control parameter, with a range $0 \le x \le 1$. The stationary state of the MVM presents a second-order phase transition, at some critical value x_c, and previous numerical works on regular lattices show that the critical exponents are compatible with its equivalent lattices for the Ising model [2–5]. Those results seem to confirm the conjecture that non-equilibrium models with up–down symmetry and spin-flip dynamics fall in the universality class of the Ising model [6]. However, numerical simulations in non-regular latices, such Archimedean, small-world or random lattices, rather seem to indicate that the critical exponents are governed by the lattice topology [7–15].

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http://dx.doi.org/10.1016/j.physa.2015.10.098
0378-4371/

On the other hand, the MVM belongs to a family of generalised spin models [16] that can be modelled in terms of a competing dynamics, induced by heat baths at two different temperatures (on two dimensional square lattices) [17,18] and hence have a *non*-equilibrium stationary state. The equilibrium stationary state of the ferromagnetic Ising model is recovered when both temperatures are equal; in this case, the detailed-balance condition is fulfilled. All members of the family share the same definition of the order-parameter and there is a critical line that separates the paramagnetic (disordered) phase from the ferromagnetic (ordered) phase [2]. So far, the ferromagnetic–paramagnetic transition out of equilibrium has been extensively studied with a single-flip dynamic rule that recovers the equilibrium Ising model. When antiferromagnetic interactions are included in the Ising model, the phenomenology becomes richer, additional phases with different critical behaviour, multicritical points, etc. can appear (see Ref. [19] and references therein). In this work, we want to explore, as a starting point, if it is possible to implement an antiferromagnetic non-equilibrium version that follows a single-flip updating scheme. Godoy and Figueiredo [20] proposed a non-equilibrium mixed-spin antiferromagnetic model with two updating schemes: one single-spin via Glauber dynamics and a two-spin updating linked to an external energy source. The aim of the present work is to implement the antiferromagnetic version of the MVM on the square and the honeycomb two-dimensional lattices, evaluate the critical point and the stationary critical exponents, β, γ and ν.

This work is organised as follows: in Section 2, the antiferromagnetic MVM is defined and the finite-size scaling method used to analyse its stationary state is outlined. In Section 3, the results of the Monte Carlo simulation are reported and the critical parameters are extracted. We conclude in Section 4.

2. Model and finite-size scaling

In the ferromagnetic version of the MVM [2], each lattice site is occupied by a spin, σ_i, that interacts with its nearest neighbours. The system evolves in the following way: during an elementary time step, a spin $\sigma_i = \pm 1$ on the lattice is randomly selected, and flipped with a probability given by

$$p(x) = \frac{1}{2}(1 + \eta x). \tag{1}$$

Herein, η stands for the (ferromagnetic) rule

$$\eta = \begin{cases} -\text{sgn}[H_i \cdot \sigma_i]; & \text{if } H_i \neq 0 \\ 0; & \text{if } H_i = 0 \end{cases} \tag{2}$$

and where H_i is the local field produced by the nearest neighbours to the ith spin. The control parameter x acts analogously to a noise in the system. With the dynamics (1) and (2), a given spin σ_i adopts the sign of H_i (the majority of its nearest neighbours) with probability $(1+x)/2$ and the opposite sign of H_i (the minority) with probability $(1-x)/2$. This evolution rule is fully equivalent to the one used in Ref. [2] with $x = 1 - 2p$ or the used in Ref. [3] with $x = \tanh(1/T)$. In the Ising ferromagnet, the sign of the bilinear exchange interaction in the hamiltonian defines if the system is ferromagnetic, or antiferromagnetic, respectively, for a positive or negative sign.

An *anti*ferromagnetic version of the MVM is now naturally obtained by replacing the rule (2) by

$$\eta = \begin{cases} +\text{sgn}[H_i \cdot \sigma_i]; & \text{if } H_i \neq 0 \\ 0; & \text{if } H_i = 0 \end{cases} \tag{3}$$

in combination with the flip probability (1), with $0 \leq x \leq 1$. The anti-ferro MVM is defined by the dynamics (1) and (3), with the control parameter $0 \leq x \leq 1$. Alternatively, one could keep the rule (2), but replace $x \mapsto -x$ in the flip probability (1). The consideration of square and honeycomb lattices allows to study the effect of having an even or odd number of nearest neighbours of each spin on the critical behaviour in the stationary state.

In analogy with simple Ising magnets, in order to measure the paramagnetic–antiferromagnetic phase transition, we shall use the *staggered magnetisation* as order-parameter

$$\langle m \rangle = \frac{1}{N} \left\langle \left| \sum_i c_i \sigma_i \right| \right\rangle, \tag{4}$$

where N is the total number of lattice sites, c_i takes values of ± 1 depending in which sub-lattice the site is located (see Fig. 1) and $\langle \cdots \rangle$ stands for time average taken in the stationary regime.

The (staggered) susceptibility is given by

$$\chi = Nx\{\langle m^2 \rangle - \langle m \rangle^2\}, \tag{5}$$

where the parameter x is included in order to have compatibility with the definition of the susceptibility in the Ising model $\frac{k_B T}{J} \chi = N\{\langle m^2 \rangle - \langle m \rangle^2\}$. We shall use the method proposed in Ref. [21], where three different cumulants are used for the evaluation of the critical point: (i) the fourth-order or Binder cumulant [22]

$$U^{(4)} = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2}, \tag{6}$$