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A new method optimizing the subgraph centrality of large networks

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HIGHLIGHTS

- We derive a better measure of network connectivity for large networks.
- A new strategy that can increase/decrease network connectivity the most is derived.
- We derive two complete functions of spectral density for two types of networks.
- An optimization algorithm based on spectral density is proposed.
- Our new findings about spectral density are also concluded.

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ABSTRACT

Since many realistic networks such as wireless sensor/ad hoc networks usually do not agree very well with the basic network models such as small-word and scale-free models, we often need to obtain some expected structural features such as a small average path length and a regular degree distribution while optimizing the connectivity of these networks. Although a minor addition of links for optimizing network connectivity is not likely to change the structural properties of a network, it is necessary to investigate the impact of link addition on network properties as the number of the added links increases. However, to the best of our knowledge, the study of that problem has not been found so far. Furthermore, two closely related questions to that problem, i.e., how to measure and how to improve network connectivity, have not been studied carefully enough yet. To address the three problems above, the authors derive a better measure of network connectivity for large networks and a new strategy that can increase/decrease network connectivity the most, and propose a spectral density algorithm optimizing the connectivity of large networks, which is able to indicate the impact on the structural properties of a network while increasing/decreasing its connectivity, providing us a guided optimization of network connectivity. In other words, our algorithm can optimize not only the connectivity of a large network but also its structural features. Meanwhile, our new findings about spectral density are also concluded in this paper. In addition, we may also apply this algorithm to solve all eigenvalues of an $N \times N$ matrix, with a low complexity of $O(N^2)$ at most.

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1. Introduction

Many complex systems can be modeled by networks to capture the possibly inhomogeneous patterns of interactions within complex systems. Due to the diversity of complex systems, the study of complex networks pervades many fields of science, such as mathematics, physics, sociology, computer science, and biology [1,2]. In recent years, there has been a great interest in the study of measuring and optimizing the robustness or resilience of complex networks, as one of the most important topics in complex networks [3,4]. To evaluate the robustness properties of complex networks, various structural and spectral measures have been proposed on the basis of different quantitative properties of underlying networks [5]. Examples are degree distribution, average path length, clustering coefficient, betweenness, global efficiency, largest component (structural measures), as well as algebraic connectivity, spectral radius (spectral measures) etc., which can be employed as the optimization objectives for promoting the robustness of complex networks.

Although structural measures represent the topological properties of a network more directly compared to spectral measures, a spectral measure of a network usually contains more abundant characteristic information of this network than a structural measure. For example, a network with a large value of algebraic connectivity, i.e. the second smallest eigenvalue of the Laplacian matrix of this network, has not only high network connectivity but also a low threshold of coupling strength for synchronization [6]. Therefore, in most cases, we prefer to make use of spectral measures while evaluating and optimizing the robustness of complex networks.

Network connectivity is a crucial form of network robustness in view of its wealthy robustness implications [3]. As mentioned above, currently algebraic connectivity acts as the prime measure that evaluates the connectivity robustness of complex networks. However, algebraic connectivity has its disadvantages to measure network connectivity while an overall network graph is disconnected, because the value of algebraic connectivity is always equal to zero for any overall disconnected network regardless of the local connectivity of this network. Consequently, in this paper we will introduce subgraph centrality [7] as another spectral measure to evaluate network connectivity and as another optimization objective to enhance network connectivity.

To improve the connectivity of an existing large real-world network, instead of substituting its infrastructure for the optimal one that maximizes its connectivity as discussed in Ref. [8], a minor modification on the current network, i.e. adding a small number of links, is usually required due to economic concerns [9]. Nevertheless, here an important question is how we can decide the exact number of the links that need adding in order to increase the connectivity of a network. For an undirected network $G(V, E)$ consisting of node set V and link set E with N nodes and L links, the number of the added links should be $\frac{N(N-1)}{2} - L$ to construct a full connected topological graph with the optimal connectivity, if we do not take into account the cost of link additions. Thus, for a large realistic (usually sparse) network, the number of the links that need adding is often bounded by the constraint on the link addition cost. However, if some links are added at an enough low cost that could be ignored, should we add an adequate number of links to a network till it becomes a full connected topology? The answer is NOT in most cases. This reason is that the construction of a full connected topology would extremely likely make the original topological properties of this network changed. For instance, a scale-free network will probably not have some topological features we are expecting any more after some additional links are included. Especially for wireless communication networks, a full connected topology will increase dramatically the MAC collisions due to the lack of its sparseness. In most cases, the optimization objectives for a large realistic network are probably not only to improve its connectivity but also to remain its structural properties unchanged even to promote its certain topological features, e.g., the degree distribution with power-law form, the heterogeneity, and the scalability, etc. Therefore, one of important aims in this paper is to optimize the topological properties of a network besides its connectivity, provided that its cost constraint allows.

To optimize the connectivity of a network, another important question is where we should add a link in this network such that its connectivity can be increased the most. The investigation on adding one link to improve network connectivity can guide us how to dynamically add a set of links one by one such that network connectivity is maximally increased. Hence, we propose a new strategy of adding a link, based on the eigenvector components of the adjacency matrix of a network, to improve its connectivity.

To the best of our knowledge, these three questions discussed above: (a) how to determine a spectral measure of network connectivity as the optimization objective, (b) how to optimize not only the connectivity of a network but also its topological features if allowed by the cost constraint, and (c) how to add a link to a network such that its connectivity can be increased the most, have not been studied intensively so far. In particular, the study of problem (b) has not been found yet. These three critical problems in network connectivity optimizations will be investigated in this paper.

The rest of this paper is organized as follows. Section 2 presents a spectral measure of network connectivity (namely subgraph centrality) and its approximation (namely spectral radius) acting as the optimization objective, instead of the conventional measure of algebraic connectivity. Section 3 presents a new strategy increasing spectral radius based on eigenvector components, and explains why it can be increased the most by using this strategy. In Section 4, we introduce the spectral density measure of network properties as the optimization objective, which is usually treated as the “fingerprint” and “pulse manifestation” of a large network. Section 5 presents the optimization algorithm of network connectivity on the basis of the spectral density of this network. Some optimization examples are given in Section 6. Section 7 presents our conclusions.