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Activity of a social dynamics model

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HIGHLIGHTS

- New measurements based on the concept of activity per agent are proposed.
- The variance of the system activity can be used to indicate the critical points of the transition.
- The frequency distribution of system activity is able to show the order of the phase transition.
- A power law dependence between cluster activity and cluster size is verified.

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ABSTRACT

Axelrod's model was proposed to study interactions between agents and the formation of cultural domains. It presents a transition from a monocultural to a multicultural steady state which has been studied in the literature by evaluation of the relative size of the largest cluster. In this article, we propose new measurements based on the concept of activity per agent to study the Axelrod's model on the square lattice. We show that the variance of system activity can be used to indicate the critical points of the transition. Furthermore the frequency distribution of the system activity is able to show a coexistence of phases typical of a first order phase transition. Finally, we verify a power law dependence between cluster activity and cluster size for multicultural steady state configurations at the critical point.

1. Introduction

The process of opinion formation has always attracted the interest of economists and political scientists [1,2]. In a democratic society where the individuals can express themselves in favour of or against a proposal, the understanding of consensus formation plays a crucial role. Typically, a consensus is reached when the majority has the same opinion or, in a specific case, votes in a particular proposal. In this context, Axelrod proposed a model to study interactions between agents in a society [3]. In the Axelrod model, an agent i is represented by a set of F independent variables, called cultural features. To each cultural feature is assigned an integer value q ($q = 1, 2, \dots, Q$) that specifies its cultural trait. An agent can only interact with its nearest neighbours and the probability $P_{i,j}$ of an agent i interacting with a neighbour j is proportional to the number of cultural features shared by them. If an interaction takes place, the interacting agents become more similar to each other and the interaction probability $P_{i,j}$ is increased.

At first sight, one expects that successive interactions lead to the formation of a homogeneous culture region in the steady state. However the occurrence of such homogeneity depends on the values of parameters F and Q. Previous studies [4–6] investigated the influence of such parameters in the formation of monocultural and multicultural patterns. In the steady state, there is a monocultural pattern (ordered state) if all agents are represented by the same set of cultural features whereas there

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is a multicultural pattern (disordered state) if two or more sets of cultural features are found among the agents. Moreover, for fixed F there is a critical value $Q_c(F)$ so that the system undergoes a transition from an ordered state (for $Q < Q_c(F)$) to a disordered one (for $Q > Q_c(F)$).

The order of this transition has been investigated in one and two dimensions [7]. According to Castellano et al. [8], on a lattice with $d = 2$ dimensions, the system undergoes a continuous phase transition for $F = 2$ and a discontinuous one for $F > 2$. It is worth reminding that any numerical simulation involving a finite size system is under finite size effects [9,10]. In addition, we may note that Binder and Landau [11] have shown that finite size effects at first-order phase transition gives origin to a metastable region. This metastability occurs around the transition point and is characterized by phase coexistence — the probability distribution of the order parameter is double-peaked.

In the present study, we propose a new measurement, named activity, to investigate the properties of Axelrod's model on the square lattice. Activity per agent is defined as the number of changes in the set of cultural features of each agent. The mean activity of the system is found to present a similar behaviour to an order parameter and shown to be useful in determination of the transition from a monocultural to a multicultural state. The probability distribution of the mean activity is found to be double-peaked in the transition point for $F > 2$ and single-peaked for $F = 2$. Our results agree with the transition points found in the literature by using the mean size of the largest cluster as order parameter. Besides, the activity map reveals there are regions on the lattice where agents are subject to a greater local pressure aligning their cultural traits. Furthermore, a power law dependence of the average activity in the cluster with its size is also verified.

2. The model

Here we study Axelrod's model on a square lattice of size L with periodic boundary conditions. One agent is placed on each site of the lattice. An agent i is represented by its cultural vector ψ_i of F components,

$$\psi_i = (\sigma_{i,1}, \sigma_{i,2}, \ldots, \sigma_{i,k}, \ldots, \sigma_{i,F}), \tag{1}$$

where $\sigma_{i,k}$ denotes its kth cultural feature ($k = 1, 2, \ldots, F$) and is specified by an index q which indicates the corresponding cultural trait ($q = 1, 2, \ldots, Q$). In the model, each cultural trait indicates one of the available choices of a cultural subject (for details, see Ref. [3]).

The system evolves from a random initial configuration (where the trait of each cultural feature $\sigma_{i,k}$ is randomly chosen among the Q possible states) according to the following steps:

(1) An agent i is randomly chosen among the $N = L^2$ agents of the lattice ($i = 1, 2, \ldots, N$).
(2) An agent j is randomly chosen among the first four adjacent neighbours of agent i (von Neumann's neighbourhood).
(3) The set Θ of n cultural features $\sigma_{i,k}$ whose traits differ from $\sigma_{j,k}$ (i.e., $\sigma_{i,k} \neq \sigma_{j,k}$ for $k = k_1, k_2, \ldots, k_n$) is identified. If Θ is an empty set then the cultural vector ψ_i remains unchanged. Otherwise, a cultural feature $\sigma_{i,k_m} \in \Theta$ is selected with probability $1/n$ and the value of σ_{j,k_m} is assigned to σ_{i,k_m} with probability $P_{i,j}$ given by

$$P_{i,j} = \frac{1}{F} \sum_{k=1}^{F} \delta_{\sigma_{i,k}\sigma_{j,k}}, \tag{2}$$

where Kronecker's delta $\delta_{\sigma_{i,k}\sigma_{j,k}}$ is equal to 0 (if $\sigma_{i,k} \neq \sigma_{j,k}$) or 1 (if $\sigma_{i,k} = \sigma_{j,k}$).

Repeating this procedure N times is counted as a Monte Carlo step. The system is let to evolve during successive Monte Carlo steps. The interaction probability $P_{i,j}$ of a pair (i, j) of adjacent agents is proportional to the number of cultural features shared by them. Successive interactions lead to a steady state configuration. Whenever $P_{i,j}$ is either equal to zero or one for every pair (i, j), the system configuration does not change anymore thus reaching an absorbent state. One or more clusters of neighbouring agents with the same cultural vector are then identified. The final configuration is either characterized by a monocultural phase (an ordered state described by the dominance of just one cultural vector) or a multicultural phase (a disordered state described by the presence of two or more cultural vectors).

In the literature [4,5], the transition between the ordered and disordered states has been studied through evaluation of the relative size S of the largest cluster ($S = N_D/N$, where N_D is the number of agents of the dominant cluster). Indeed the mean value $\langle S \rangle$ (averaged over M simulations) has been taken as the order parameter of the system.

However this order parameter is not able to show how the local pressure affects the agents as the system evolves. According to Axelrod [3], the formation of an ordered state is due to mechanisms of local convergence which in turn can also lead to a disordered state. To elucidate the role of the local pressure and local convergence in the dynamics, we propose the concept of activity to study the system.

3. Activity

We define the activity per agent at time step t as

$$a_i(t) = \frac{N_{\psi_i}(t)}{t}, \tag{3}$$

where $N_{\psi_i}(t)$ is the number of changes in the cultural vector ψ_i during the first t Monte Carlo steps. At each time step, an agent i has probability $1/N$ of being randomly chosen and probability $P_{i,j}$ of interacting with some neighbour j. Since N