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# A synergetic model for human gait transitions

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## HIGHLIGHTS

- A synergetic model for human gait transitions is presented.
- Two experiments were conducted to study human gait transitions on a treadmill.
- The effect of acceleration and inclination on gait transitions were investigated.
- The results of the experiments have been described using the proposed model.

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## ABSTRACT

Gait transitions have been considered as bifurcations between states (e.g. walking or running modes) of a nonlinear dynamical system. A top-down synergetic approach to model gait transitions has been adapted from Frank et al. (2009) and applied to two sets of empirical observations. In this approach, it is assumed that the amplitudes of the spatio-temporal modes of locomotion satisfy a generic form of evolution equations that are known to hold for animate and inanimate self-organizing systems. The presented experimental results focus on hysteresis in human walk-to-run and run-to-walk transitions on a treadmill as a function of treadmill inclination and acceleration, the rate at which speed was increased or decreased during experimental trials. The bi-stability in the synergetic model is assumed to account for the hysteretic transitions. Accordingly, the relevant parameters of the model were estimated from the empirical data and the model's efficacy in predicting the observed hysteresis effects was evaluated.



## 1. Introduction

Spontaneous switching between behaviors is commonplace. For example, bipeds and quadrupeds switch from walking to running or vice versa depending on their required locomotion speed. In a laboratory setting, specifically on a treadmill with varying speed, the transition between gaits (walking or running) has been investigated extensively. It is known that the transition depends on different parameters such as speed, stride length and stride frequency [1].

The switching between behaviors is based on the perception of an opportunity for action in the environment (e.g., running) that one perceives relative to one's dimensions and action capabilities (e.g., leg length, weight, etc.). This idea that the environment is perceived as body-scaled reflects Gibson's concept of *affordances* [2]. For example, a treadmill operating at a particular speed that affords walking for an adult (with longer legs) can only afford running for a child (with shorter legs). In some studies, it has been considered that a dimensionless ratio, known as a $\pi$-number [1,3], invariantly specifies the boundary between walking and running irrespective of different action capabilities of differently sized individuals.

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In this context it has been proposed that locomotory movements are dynamically similar when their Froude ($Fr$) numbers are equal (e.g., Ref. [4]), where $Fr = v^2/(lg)$ (i.e., the squared speed, $v$, divided by the product of leg length, $l$, and gravitational acceleration, $g$). The $\pi$-number $Fr$ can be characterized as the ratio of inertial to gravitational forces. In what follows, the $Fr$ number at which the transition occurs will be the critical $Fr$ number.

The physical interpretation of critical $Fr$ is that the walk-to-run transition occurs when the acceleration of the body's center of gravity resulting from inertial forces exceeds the acceleration caused by gravity. In several studies, it has been shown that the transition as measured by the critical Froude number differs experimentally for systematically increasing and systematically decreasing the $\pi$-number or the speed: there is *hysteresis* (e.g. Refs. [1,5–7]). Hysteresis is defined as a property of a system such that an output value is not a strict function of the corresponding input, but also incorporates some delay or history dependence. In particular, hysteresis refers to the case when the response for a decrease in the input variable is different from the response for an increase. Hysteresis has been identified in various psychological phenomena and in particular in affordance research (e.g. in perceptual speech categorization in Tuller et al. [8]; in perceptual judgments of the maximum slope of a surface that afforded upright posture with either visual or haptic exploration in Fitzpatrick et al. [9]; in perception of maximum sit-on-ability in Hirose et al. [10]; in perception of grasp-ability of objects in Richardson et al. [11]). The hysteresis phenomenon in gait transition – higher transition speed for walk-to-run (WR) than for run–walk (RW) – has also been reported in a number studies (e.g. Refs. [1,7]).

The aim of the present work is to study the effects of biomechanical constraints on hysteresis using a synergetic dynamical model. To date, there is very little effort in mathematical modeling of gait transition (for an exception see e.g. the probabilistic model by Li [12]). The dynamical model used in this study is adapted from the grasping transition (GT) model in Frank et al. [13]. The GT model [13] was originally derived from a model proposed by Haken [14]. Haken's model was aimed to describe pattern recognition and oscillatory perceptual processes of ambiguous patterns [15]. The model proposed by Haken, which has often been interpreted as a neural network model (see Refs. [16,17]), assumes that attractors and repellers of the neural system of humans and animals determine states of neural activity. In the context of behavioral transitions, these are the attractors and repellers of the organism–environment system that determine states of behavior. For instance, in grasping transition behavior, the information about the to-be-grasped objects (i.e. object size) and the relevant properties of the individual's action system (i.e. hand span) constrains and guides the grasping behavior of an individual.

### 1.1. Synergetic model

Historically, gait transitions are considered to correspond to bifurcations between the states of a nonlinear dynamical system (e.g. Refs. [18–21]). The behavior of such a system can be described in terms of an *order parameter*, which can be the relative phase between limbs or limb segments. Continuous variation of a *control parameter*, e.g. treadmill's speed, can induce bifurcations in the order parameter [20].

The mathematical theory of synergetics assumes that the behavior of a system is decomposable into the time-evolution of its dynamical modes [14]. A system's dynamical mode is characterized by an eigenvector and an eigenvalue. An eigenvector corresponds to an observed mode, behavior, or pattern. An eigenvalue describes the rate of change of a dynamical mode. Note that we will re-interpret below the eigenvalue as *availability parameter*.

In the context of human WR or RW gait transitions, we assume $\xi_1(t)$ (or $\xi_1$) and $\xi_2(t)$ (or $\xi_2$) represent the dynamical evolution of generalized amplitudes of the walking and running modes, respectively.[1] Then, as in Frank et al. [13], we assume the following potential function for the system with the two dominant modes:

$$V(\xi_1, \xi_2) = -\frac{1}{2}\left(\lambda_1\xi_1^2 + \lambda_2\xi_2^2\right) + \frac{b}{2}\left(\xi_1^2\xi_2^2\right) + \frac{c}{4}\left(\xi_1^2 + \xi_2^2\right)^2. \tag{1}$$

When $\xi_1 > 0$ and $\xi_2 = 0$, walking ($Y^W$) is performed and when $\xi_2 > 0$, $\xi_1 = 0$, running ($Y^R$) is performed. In Eq. (1), $\lambda_1$ and $\lambda_2$ are availability parameters defining the possibilities for walking and running, respectively. The magnitude of $\lambda$ determines how strongly one of the modes is activated. A behavioral mode is available for $\lambda > 0$ and is not available for $\lambda < 0$.[2] Fig. 1 depicts the potential function with constant coefficients $b$ and $c$. For $\lambda_1 > 0$ and $\lambda_2 < 0$, the potential shows minima on $\xi_2 = 0$ axis indicating that the walking mode is the only stable mode (Fig. 1; left panel). For $\lambda_1 < 0$ and $\lambda_2 > 0$, the potential includes minima on $\xi_1 = 0$ axis explaining that the running mode is the stable mode (Fig. 1; center panel). For $\lambda_1$ and $\lambda_2 > 0$, depending on the boundary conditions at $[\xi_1(0), \xi_2(0)]$, both walking and running modes can be stable (Fig. 1; right panel).

Dynamical evolution of amplitudes of the system's modes is governed by the following equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\xi_1 = -\frac{\partial V(\xi_1, \xi_2)}{\partial \xi_1} \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\xi_2 = -\frac{\partial V(\xi_1, \xi_2)}{\partial \xi_2}. \tag{3}$$

[1] Note that spatial characteristic of the modes can remain non-specified in this approach. In other words, magnitudes of $\xi_1$ and $\xi_2$ are not directly evaluated.

[2] Although for $\lambda > 0$ a mode is available, the mode can be either stable or unstable. Only stable available modes are performed. In general, if $\lambda_1$ (or $\lambda_2$) is much larger than $\lambda_2$ ($\lambda_1$), then $Y^W$ (or $Y^R$) is stable. For $\lambda_1 = \lambda_2$, both modes are stable.