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Random matrix theory and portfolio optimization in Moroccan stock exchange

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HIGHLIGHTS

- We studied the cross-correlation among stocks of Casablanca Stock Exchange portfolio.
- We used Marčenko–Pastur distribution to analyze eigenvalues.
- We analyzed distribution of eigenvectors components.
- We used the inverse participation ratio to measure the deviation degree of eigenvectors.
- We found that more than 11% of eigenvalues might contain the pertinent information.

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ABSTRACT

In this work, we use random matrix theory to analyze eigenvalues and see if there is a presence of pertinent information by using Marčenko–Pastur distribution. Thus, we study cross-correlation among stocks of Casablanca Stock Exchange. Moreover, we clean correlation matrix from noisy elements to see if the gap between predicted risk and realized risk would be reduced. We also analyze eigenvectors components distributions and their degree of deviations by computing the inverse participation ratio. This analysis is a way to understand the correlation structure among stocks of Casablanca Stock Exchange portfolio.

1. Introduction

Due to financial globalization, markets become more and more connected and dynamic. So, investors should use methods that allow them to maximize their expected returns in these markets. There exist numerous methods for this end, the most known is Markowitz's model [1]. It estimates risk and expected returns based on the standard deviation and the expected value of returns. Other methods have aroused to manage and optimize portfolios. Correlation seems to be an important element to study portfolio management; we should have a way to understand interactions among matrices of returns. Numerous methods were proposed to study cross-correlation among series [2–6]. Besides, cross-correlation was also studied among several financial series [6–13].

In this paper, we are interested on another interesting method called Random Matrix Theory (RMT) to study cross-correlations among stocks of one portfolio. This method was used in nuclear physics by Wigner [14]. It was also used by Dyson and Mehta [15] to explain the energy levels of complex nuclei [16].

RMT has been used to analyze correlation in the finance area and specially to improve portfolio management. By using RMT, Pafka and Kondor [17] found that the effect of noise in correlation matrices determined from financial series can indeed

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be large that the filtering based on random matrix theory is particularly powerful in this respect. Laloux et al. [18,19] found that the empirical correlation matrix leads to a dramatic underestimation of the real risk, by overinvesting in artificially low-risk eigenvectors. They found that less than 6% of the eigenvectors, which are responsible of 26% of the total volatility, appear to carry some information. Pafka and Kondor [20] found that "realized" risk is a good proxy for "true" risk in all cases of practical importance and that "predicted" risk is always below, whereas "realized" risk is above the "true" risk. Using a simulation-based approach they show that for parameter values typically encountered in practice the effect of noise on the risk of the optimal portfolio may not necessarily be as large as one might expect. Wang et al. [21] investigated the statistical properties of cross-correlations in the US stock market. They found that the DCCA coefficient method has similar results and properties with Pearson's Correlation Coefficient, such as the properties of the largest eigenvalue and the corresponding eigenvector. Daly et al. [22] found that RMT-based filtering can, in the most cases, improve the realized risk of minimum risk portfolios. Plerou et al. [16] analyzed cross-correlations between price fluctuations of different stocks using methods of random matrix theory (RMT). They concluded that the deviating eigenvectors are useful for the construction of optimal portfolios that have a stable ratio of risk to return.

In this paper, we use Random Matrix Theory to study cross-correlation among stocks of Casablanca Stock Exchange. We clean the correlation matrix to observe if the difference between predicted risk and realized risk will be reduced. We also analyze eigenvectors through their distributions and by computing the inverse participation ratio. This paper is organized as follows, in Section 2 we present a brief description of data. Then, we expose in Section 3 the theoretical background of RMT. In Section 4, we show the main empirical results and finally we conclude.

2. Data

The data used include 62 securities listed in the Casablanca Stock Exchange.[1] We chose the period from the 1st January 2008 to the 3rd January 2014, we have then 1492 daily closing prices and 1491 logarithmic returns.

We designate by p_t the closing price of the index on day t. In the present paper, the method applied to the natural logarithmic returns of the index is defined by:

$$r_t = \ln\left(\frac{p_{t+1}}{p_t}\right). \tag{1}$$

Then, we compute mean return and standard deviation of each of these securities before establishing the portfolio selection process.

3. Theoretical background

In order to quantify correlations, we first calculate the price change ("return") of stock $i = 1, \ldots, N$ over a time scale Δt,

$$G_i(t) \equiv \ln S_i(t + \Delta t) - \ln S_i(t),$$

where $S_i(t)$ denotes the price of stock i. Since different stocks have varying levels of volatility (standard deviation), we define a normalized return

$$g_i(t) \equiv \frac{G_i(t) - \langle G_i\rangle}{\sigma_i},$$

where $\sigma_i \equiv \sqrt{\langle G_i^2\rangle - \langle G_i\rangle^2}$ is the standard deviation of G_i, and $\langle \ldots \rangle$ denotes a time average over the period studied. We then compute the equal-time cross-correlation matrix C with elements

$$C_{ij} \equiv \langle g_i(t) g_j(t)\rangle.$$

By construction, the elements C_{ij} are restricted to the domain $-1 \le C_{ij} \le 1$, where $C_{ij} = 1$ corresponds to perfect relations, $C_{ij} = -1$ corresponds to perfect anti-correlations, and $C_{ij} = 0$ corresponds to uncorrelated pairs of stocks.

The difficulties in analyzing the significance and meaning of the empirical cross-correlation coefficients C_{ij} are due to several reasons, which include the following:

(i) Market conditions change with time and the cross-correlations that exist between any pair of stocks may not be stationary.
(ii) The finite length of time series available to estimate cross-correlations introduces "measurement noise".

If we have N returns with the same length equal to L, then, the empirical cross-correlation matrix $\mathbf{C}$ could be computed by C_{ij}. In our case, we have $N = 62$ and $L = 1491$. By diagonalizing matrix $\mathbf{C}$, we obtain

$$\mathbf{C}\mathbf{u}_k = \lambda_k \mathbf{u}_k.$$

In matrix notation, the correlation matrix can be expressed as

$$C = \frac{1}{L} G G^T$$

[1] http://www.casablanca-bourse.com/.