Physica A 433 (2015) 229–246

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Stochastic string models with continuous semimartingales

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HIGHLIGHTS

- We reformulate the stochastic string framework using continuous semimartingales.
- We obtain the dynamics of the short-term interest rate and a PDE for bond prices.
- We present an analytic expression for the price of a European bond call option.
- We show that the stochastic string model is relevant for derivative pricing.
- We show that the stochastic string model is equivalent to an infinite dimensional HJM model to price European options.

ARTICLE INFO

Article history:
Received 6 November 2014
Received in revised form 26 January 2015
Available online 1 April 2015

Keywords:
Semimartingale
Stochastic string
Derivatives pricing
Term Structure
Random field
Quantum field

ABSTRACT

This paper reformulates the stochastic string model of Santa-Clara and Sornette using stochastic calculus with continuous semimartingales. We present some new results, such as: (a) the dynamics of the short-term interest rate, (b) the PDE that must be satisfied by the bond price, and (c) an analytic expression for the price of a European bond call option. Additionally, we clarify some important features of the stochastic string model and show its relevance to price derivatives and the equivalence with an infinite dimensional HJM model to price European options.

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1. Introduction

Among the continuous-time models of the term structure of interest rates (TSIR), the model of Heath, Jarrow, and Morton [1] (HJM, hereafter) is considered by many as one of the main references. In this model, the initial forward curve and the volatility structure of forward rates are taken as inputs so that we can determine the evolution of the forward curve over time from the dynamics of the instantaneous forward rate and the no arbitrage condition. The main advantages of this model are (see Ref. [2]): (a) a perfect fitting (by construction) to the initial TSIR, (b) its flexibility, that allows to obtain different models for different volatility functions, and (c) its generality as it nests several models.[2] Additionally, the one-factor version of this model is simple, parsimonious, and consistent with some empirical evidence (see Ref. [9]).

However, a drawback of this model is that, for a given volatility structure, the one-factor version imposes a perfect correlation between the innovations of instantaneous forward rates for different maturities. This restriction limits the

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1 The usual caveat applies.

2 See, for instance, Refs. [3–8].

http://dx.doi.org/10.1016/j.physa.2015.03.070
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dynamics of the forward curve and contradicts the empirical evidence (see Ref. [10]), that shows that the correlation decays exponentially as a function of the difference between maturities. Merton [11] shows a no-arbitrage relationship between the price of a portfolio of options and the price of an option on a portfolio, determined by the correlation between the underlying assets. Then, to perform the relative pricing of caps and swaptions, the TSIR model must reflect the correlation between forward rates more appropriately than one-factor models, which do not allow a fast enough decorrelation (see Refs. [12–14]) nor an independent description of volatilities and correlations (see Refs. [15,16]). A solution could be increasing the number of factors to consider imperfect correlation but the theoretical volatilities corresponding to each factor must be parameterized to fit simultaneously empirical volatilities and correlations, a really difficult goal (see Ref. [17]).

A new type of TSIR models, known as *stochastic string* or *random field* models, has proposed a correlation structure between forward rates. Seminal papers of this string approach are Refs. [17–20], which are followed by Refs. [15,21–25], among many others.[3]

All these papers consider that the source of randomness generating the dynamics of the forward curve, the *stochastic string process*, is not constant along maturities but it varies point by point along the whole TSIR. The only condition is that the shocks with different maturities must be imperfectly correlated but maintaining the continuity in the forward curve. As a consequence, the string models provide several advantages with respect to the traditional HJM ones, as pointed out by Refs. [17,20,25,26,29], among others.

Considering these advantages, it seems reasonable to look for a TSIR framework as general as possible. Santa-Clara and Sornette [17] is the paper closest to this goal. However, from our point of view, that paper lacks of rigorous mathematical formulation to be the benchmark we are looking for. This lack is recognized by these authors, who state: "We attempt to present the results and their derivation in the simplest and most intuitive way, rather than emphasize mathematical rigor" (see Ref. [17, p. 150]).

Our main objective is to provide a framework, based on stochastic calculus, that can incorporate existing formulations of string models of the TSIR and that, at the same time, is able to accommodate future extensions of these models. To this aim, we rebuild the model of Santa-Clara and Sornette [17] and provide some new results not included in that paper.

Our first contribution is that we characterize clearly the type of stochastic integration to be used, an issue omitted in Ref. [17]. We will use semimartingales, the more general processes for which we can define reasonably a stochastic calculus (see Ref. [30]). Moreover, in arbitrage-free models, the price of any financial asset follows a semimartingale (see Refs. [31,32]).

The second contribution of the paper is related to the methodology. Santa-Clara and Sornette [17] consider a certain *stochastic discount factor* and impose that the bond price (discounted using this factor) follows a martingale. Our approach follows the more standard methodology in Finance of discounting prices by means of a banking account.

Our final contribution is that we obtain certain original results with respect to Ref. [17] such as: (a) an expression for the dynamics of the short-term interest rate, (b) a Markovian framework in which we will obtain a PDE to price bonds, and (c) a closed-form expression for the price of European bond options in the Gaussian case.

This paper is organized as follows. Section 2 details the probabilistic framework and introduces the financial concepts and mathematical results to be used later. Section 3 generalizes the stochastic string process of Ref. [17] to a framework based on calculus with semimartingales. An alternative way of building examples of stochastic string processes using the random field theory is reported. Section 4 provides the dynamics of the variables that characterize the TSIR and the no-arbitrage condition of the model and shows that the one-factor HJM model is nested in the stochastic string one. Section 5 includes the conditions under which the short-term interest rate follows a Markovian process and, as an application, we obtain a bond pricing PDE that recovers some of the classical TSIR models.

To illustrate the usefulness of the model, Section 6 prices analytically European bond options in the Gaussian case, generalizing the formulas obtained for the Gaussian HJM models. Section 7 analyzes the relationship between our model and another ones proposed in the literature and provides some interesting properties on the robustness of the stochastic string models. Finally, Section 8 summarizes the main conclusions. Mathematical proofs are deferred to the Appendix.

2. Notation and preliminary results

We consider a filtered complete probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ satisfying the usual hypotheses (see Ref. [33, p. 3]). We also assume that $\mathcal{F} = \mathcal{F}_T$ where T denotes the finite time horizon for trading risk-free zero-coupon bonds. All the equalities and properties related to stochastic processes will be assumed to hold almost sure.

Let P_t^τ be the price at time t of a bond maturing at time τ, where $0 \leq t \leq T$ and $t \leq \tau$. For each fixed $\tau \geq 0$, we have a real stochastic process

$$\begin{aligned} P^\tau &: [0, \tau] \times \Omega \to \mathbb{R} \\ (t, \omega) &\longmapsto P_t^\tau(\omega) \end{aligned}$$

verifying $P_\tau^\tau = 1$, $P_t^\tau > 0$, and that $\frac{\partial \ln P_t^\tau}{\partial \tau}$ exists for all t.

[3] See also Refs. [16,26–28].

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