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# Efficiency driven evolution of networks

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## HIGHLIGHTS

- We suggested that transportation network evolves to increase its efficiency.
- A possible measure of the efficiency is proposed.
- Numerical results on the evolution of efficiency are given.
- Some properties of the spectrum of the network are discussed.

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ABSTRACT

A principle governing the evolution of network is proposed based on the consideration of improving the efficiency of a network by demanding that new connections should be added in the network to increase the transportation efficiency as much as possible. It turns out that the added connections should be between two unconnected nodes with sum of their links maximum. The evolution of some properties of the network is considered, such as the degree distributions, number of shortest paths with different distances, and the moments of the spectrum distributions.



## 1. Introduction

The complex network has been attractive in recent a few decades not only as the pattern discovered ubiquitously in real world, but also as the unifying framework to understand inherent complexity in nature. It has been shown that many social [1,2] and natural [3] phenomena can be described based on models of complex networks. To understand the different features of the networks, many mechanisms have been proposed for the formation and evolution of the networks, such as the random generation of networks [4], rewiring from a regular network to form the small world networks [5] and the preferential attachment for the generation of the scale-free networks [6]. Those models for the generation and/or evolution of networks have little to do with the underlying physical or economical constraints. In real world, though, such constraints play important roles. For example, a rail way between two cities cannot be built if there is no investment from government or some company. Some researches focus on improving the service of transportation network in a city by reducing the number of transfers and the total time of the users [7]. In those researches, route design and operation cost are concerned, and the geometrical of nodes and corresponding population distributions are needed.

This paper is a try to investigate how to improve the connection efficiency of a network by adding new edges to the network. No economical effect is considered, and no information is introduced about the spatial distances among nodes in the network. Thus the focus of this paper is very different from that in Ref. [7]. An arbitrary transportation network is taken as an example, since in such a network, the efficiency can be more easily defined. Then the nodes can be bus stations for

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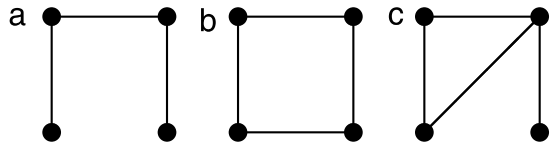


**Fig. 1.** Evolution from a simple network with only four nodes from (a) to graphs (b) and (c) by adding one edge.

the public transportation network in a city or airports for a flight network, and an edge is a bus route or a flight between two cities. For simplicity, the network is assumed to be singly connected and no multiple connection between nodes is considered. The number of nodes in the network is fixed in the investigation, because in real situations, the number of cities or towns in the consideration of a transportation network is known and fixed for quite a long time. The first question one may ask is how to measure the efficiency of a transportation network. Then one may further ask how to improve the transportation network. Since no edge rewiring is considered in the development, the improvement can be achieved only by adding new connections to the network. The problem is then where to put the newly added edge to the network. By the way, one can study some properties of the network and their evolution.

This paper is organized as follows. In next section, the efficiency of a transportation network will be discussed from usual conventional wisdom of normal people based on the number of transfers for traveling around the network. Then the optimism evolution of the network is suggested. In Section 3, some properties and their dependence on the evolution of the network will be discussed. The last section is for a brief summary and discussion.

## 2. Efficiency of a transportation network

The purpose of developing a transportation network is for the convenience of people's traveling from place to place. Because of some reasons or others, one cannot expect a direct trip between any two sites. Here by "direct trip" it means that no transfer is needed. Or in the words of network, a "direct trip" is possible between two nodes (bus stations or airports) when there is an edge connecting them. For airline network, one can go from one city to another by a direct flight or through a few connection transfers. Of course, most people enjoy fewer transfers in traveling. Therefore, to make a transportation network more convenient and more efficient, the numbers of short paths in the network should be large and those for long paths should be small. To make a quantitative measure, one can define a series of connecting matrices, the $d$-separation connecting matrices $w_{ij}^{(d)}$, for $d = 1, 2, \ldots$, whose element $w_{ij}^{(d)}$ is 0 if there is no shortest path from node $i$ to $j$ with $d-1$ transfers, otherwise it equals the number of such routes. For $d = 1$, $w_{ij}^{(1)} = w_{ij}$ is the adjacent matrix of the network which equals one if nodes $i$ and $j$ are connected by an edge, otherwise it is 0. Then the numbers $N_d$ of shortest routes in a network with separation $d$ can be easily calculated as $N_d = \sum_{ij} w_{ij}^{(d)}/2$. By definition, $N_1$ equals the number of edges in the network. With a step function $\Theta(x) = 1$ for $x > 0$ and 0 otherwise, it is obvious that for $d > 1$

$$w_{ij}^{(d)} = \left(w^d\right)_{ij} \prod_{l=1}^{d-1} \Theta(1 - w_{ij}^{(l)}), \tag{1}$$

where $(w^d)_{ij}$ is an element of the $d$th power of the adjacent matrix and the factor $\prod_{l=1}^{d-1} \Theta(1 - w_{ij}^{(l)})$ excludes possibilities that there is at least one path between nodes $i$ and $j$ with length shorter than $d$. Matrices $w^{(d)}$ and numbers of routes $N_d$ with different separations can be calculated directly from the adjacent matrix $w_{ij}$. For $N_2$, only information of the adjacent matrix $w_{ij}$ is needed. Since $w_{ij}$ can take only two values, 0 and 1, $\Theta(1 - w_{ij}) = 1 - w_{ij}$. Then one can derive a simple expression of $N_2$ from the adjacent matrix as

$$N_2 = \frac{1}{2} \sum_{ijl} \left(w_{il} w_{lj} - w_{ij} w_{il} w_{lj}\right) = \sum_{l} k_l(k_l - 1)/2 - 3C_3, \tag{2}$$

where $k_l$ is the degree of node $l$ and $C_3$ is the number of circles in the network consisting of three connected nodes.

Now one can turn to answering the question of how to increase the efficiency of a transportation network. For this purpose, one or more connections between nodes should be added. In this paper, edges are added one by one. As discussed earlier, the number of nodes is fixed during the process. It can be imagined that the efficiency may be different if an edge is added to a network between different nodes. The question one should answer is where one should put an edge to the network for optimizing the efficiency increase. To get some hint, one can have a look at a simple network, as shown in Fig. 1(a). The network consists of four nodes and three edges. One can add one edge to the network in three ways. One way is shown in Fig. 1(b), and two other ways can be shown by Fig. 1(c). One can see the difference in efficiency increase from Fig. 1(a) to Fig. 1(b) or Fig. 1(c) by counting the numbers of routes with different separations, which are tabulated in Table 1. With the addition of one edge, $N_1$ increases by one in the two cases and $N_3$ decreases to zero, while in Fig. 1(c) $N_2$ is smaller than in Fig. 1(b). From the table, one learns that one can travel through the network in Fig. 1(c) by 4 paths with separation 1 and 2 paths with separation 2. In Fig. 1(b) the number of paths with separation 2 is bigger by 2 than in Fig. 1(c), indicating