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Cascading failures in interconnected networks with dynamical redistribution of loads

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HIGHLIGHTS

- We model an interconnected network based on dynamical redistribution of flows.
- Enhancing the heterogeneity will make networks more susceptible.
- The coupling preference makes almost no differences to the resilience.
- A model of interconnected traffic networks of Beijing city is analyzed.

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ABSTRACT

Cascading failures of loads in isolated networks and coupled networks have been studied in the past few years. In most of the corresponding results, the topologies of the networks are destroyed. Here, we present an interconnected network model considering cascading failures based on the dynamic redistribution of flow in the networks. Compared with the results of single scale-free networks, we find that interconnected scale-free networks have higher vulnerability. Additionally, the network heterogeneity plays an important role in the robustness of interconnected networks under intentional attacks. Considering the effects of various coupling preferences, the results show that there are almost no differences. Finally, the application of our model to the Beijing interconnected traffic network, which consists of a subway network and a bus network, shows that the subway network suffers more damage under the attack. Moreover, the interconnected traffic network may be more exposed to damage after initial attacks on the bus network. These discussions are important for the design and optimization of interconnected networks.

Since the end of the last century, with the introduction of small-world networks and the scale-free networks, the study of network science has attracted an increasing amount of attention. Currently, network theory has become one of the major tools for studying systems engineering and complexity science, and it has profound guiding significance for every aspect of human action.

The study of cascading failures, an important research area of complex networks, has proved useful in various fields, especially in aiding and ensuring the normal function of critical infrastructures such as power grids and communication networks [1–10]. In the past decade, most of these studies were based on the assumption that networks are isolated, neglecting the fact that modern systems are coupled together [11–15]. To study the impact of coupling in coupled networks, a theory of interdependent networks was presented [16–20]. In Ref. [16], a mutual percolation model was proposed by Buldyrev et al. to study the vulnerability of network of networks where the links between the networks are interdependent. Zhang et al. extended the interdependent network model by considering flows in the networks, showing that the robustness

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of interdependent scale-free (SF) networks is much smaller than single SF networks or interdependent SF networks without flows. In addition, the authors illustrated that a valley of minimal network robustness exists within a range of scaling exponents [21]. In addition to interdependent networks, the interconnected network is another type of coupled networks. It is available in such situation as new transmission lines introduced among existing separated power grids [22,23]. Brummitt et al. [23] indicated that adding connectivities between two isolated networks is beneficial to suppress the largest cascades in each system. However, increasing the number of interconnectivities becomes detrimental. In Ref. [24], Tan et al. discussed the effect of the coupling probability and the coupling preference of cascading failures in interconnected networks. The authors found that assortative coupling is more helpful for resisting the cascades than are disassortative or random coupling.

In the work mentioned above, the topology of the networks was altered during the propagation of the failures. However, in reality, many societal studies suggest that components cannot be added or removed from networks. For instance, the bus network and the subway network are coupled by interconnected links, which provide paths for transfer between them. The distribution of flows on them, which will not shut down any stations from the whole network, should be drawn attention especially. Crucitti et al. presented a model to illustrate how the breakdown of a single node is sufficient to collapse the entire system simply because of the dynamics of flow redistribution in the network [25]. Here, we generalize this model to interconnected networks and study its robustness against cascading failures under various attack strategies.

In this paper, we first introduce how the flow redistributes on interconnected networks in our model. Secondly, we study the robustness of interconnected scale-free (SF) networks and interconnected Erdős–Rényi (ER) networks under various attacks. We also explore the dependence of the robustness on the coupling preferences in detail. Finally, we present an application of our work on the interconnected traffic networks of Beijing city.

1. Modeling

In most of modern systems, transferring flows is a major function of them, such as electric power grids transferring currents, the Internet transferring information, and traffic networks transferring people. The transmitting capacity of a network can be measured by its "load", which quantifies the amount of flows that a node is requested to transmit and is considered to depend on the total number of shortest paths passing through it [26–28]. Therefore, the load of node i is denoted as follows:

$$L(i) = \sum_{(j,k)} \frac{\sigma_{jk}(i)}{\sigma_{jk}} \quad i = 1, 2, \ldots N \tag{1}$$

where σ_{jk} is the total number of the shortest paths between node j and k, $\sigma_{jk}(i)$ is the number of the shortest paths between node j and k through node i, and i.e. is the load of node i that equals the betweenness centrality.

Generally, in artificially created systems, designers have designed a higher node capacity to ensure that every unit operates normally; therefore, the capacity C is defined as the maximum load that the node can handle [1]:

$$C(i) = (1+\alpha)L_0(i) \quad i = 1, 2, \ldots N \tag{2}$$

where $\alpha \geq 0$ is the tolerance parameter of the network and $L_0(i)$ is the initial load of node i.

Initially, the network is in a free flow state, in which the load L at each node is lower than its capacity C. The removal of one or more nodes will result in a redistribution of loads, which may trigger failures on other nodes because the load exceeds the capacity. In Ref. [25], Crucitti et al. assumed that the currents/information/people will take the most efficient transfer path, and that the failed nodes are congested. Here, the efficiency of the most efficient path between i and j is denoted by $\frac{1}{d_{ij}}$, where d_{ij} is the length of the shortest path between any pair of nodes i and j. When a node is congested, the efficiency of all of the arcs passing through it will reduce. Eventually, the currents/information/people will take a new most efficient path. Based on the above assumptions, the relative size of the largest connected component should no longer be used to quantify the network robustness. Instead, we use the global efficiency E to quantify the network robustness [29].

$$E = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i>j} \frac{1}{d_{ij}}. \tag{3}$$

If i and j are not in a connected component, d_{ij} is ∞.

In our model, we present a weighted, undirected interconnected network consisting of networks A and B, and the initial weights of all edges are set to 1.0. These networks have the same size and form a one-to-one bidirectional connected relation. Initially, we simulate the attack of a node by removing it. Two strategies are considered: 1. Attack on a node randomly. 2. Attack on the node with the highest load. We assume that a node in network A is removed (a schematic plot is shown in Fig. 1). With the redistribution of flows in the remaining nodes, a series of nodes ($A3$ and $A5$, collectively called S_A in Fig. 1(b)) will be congested; therefore, the global efficiency of network A decreases. Meanwhile, the flows that S_A cannot handle (named LC_A, $LC_A = L_A - C_A$), will be transferred to network B through the coupling edges. Then, one or more nodes may become congested, resulting in cascading failure on network B. The flows LC_B which congested nodes ($B1$ in Fig. 1(c) are collectively called S_B) cannot handle will amplify the congestion on the network A. Due to this mutual interconnection, the load between network A and B follows the principle below:

$$L_{B_i}(t+1) = L_{B_i}(t) + [L_{A_i}(t) - C_{A_i}] \tag{4}$$

where A_i is the congested node in network A at time t and B_i is the corresponding coupled node in network B.