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Simple stochastic order-book model of swarm behavior in continuous double auction

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HIGHLIGHTS

- We present a simple stochastic order-book model for investors' swarm behaviors.
- The data obtained from one of our models shows a phenomenon called "fat tail".
- This is because orders temporarily swarm following past price trends.
- This fact is demonstrated by simulation of our follower type order-book model.

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ABSTRACT

In this study, we present a simple stochastic order-book model for investors' swarm behaviors seen in the continuous double auction mechanism, which is employed by major global exchanges. Our study shows a characteristic called "fat tail" seen in the data obtained from our model that incorporates the investors' swarm behaviors. Our model captures two swarm behaviors: one is investors' behavior to follow a trend in the historical price movement, and another is investors' behavior to send orders that contradict a trend in the historical price movement. In order to capture the features of influence by the swarm behaviors, from price data derived from our simulations using these models, we analyzed the price movement range, that is, how much the price is moved when it is continuously moved in a single direction. Depending on the type of swarm behavior, we saw a difference in the cumulative frequency distribution of this price movement range. In particular, for the model of investors who followed a trend in the historical price movement, we saw the power law in the tail of the cumulative frequency distribution of this price movement range. In addition, we analyzed the shape of the tail of the cumulative frequency distribution. The result demonstrated that one of the reasons the trend following of price occurs is that orders temporarily swarm on the order book in accordance with past price trends.

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1. Introduction

Numerous stocks and derivatives are traded on exchanges around the world. For the traders, it is crucial to estimate the profitability and downside risk of financial products they trade in. In the past, miscalculation of price movements has caused many global financial crises. Long-Term Capital Management, a hedge fund management firm, is one of the companies that has caused such crises when it suddenly incurred enormous loss and recapitalized in 1998. One of the reasons for its recapitalization was because the prices of financial products substantially changed unexpectedly. R.N. Mantegna and H.E. Stanely have pointed out, prices actually do not ideally move following a normal distribution [1]. In reality, large-scale price movement more frequently occurs than that assumed by a normal distribution. Various financial markets have experienced large-scale price movement that does not fit a normal distribution, and studies of market crashes are flourishing [2].

In this study, we studied large-scale price movements in the exchange market caused by investors' collective behaviors. Collective behaviors can be seen in various settings including economic and biological as well as other social setting. In recent years, this collective behavior has been termed "swarm intelligence". These behaviors have been extensively studied as phenomena that show a high degree of motion as a group through simple local interactions between individuals [3]. As pioneering research on collective behavior in financial markets, T. Lux and M. Marchesi explored models to reproduce phenomena, such as fat tails and the temporal independence of volatility, with regard to the interaction of strategic investor groups [4]. The present study focuses on how local investors affect each other and produce large-scale price fluctuations as a group. As such, we herein denote investors' collective behavior as "swarm behaviors". We think one of the factors of large-scale price movement is connected with certain swarm behaviors of investors. For example, once the price begins to continuously decline, investors fear downside risk, and they follow the trend by lowering the prices of their orders. This is one of the mechanisms by which price decline causes further declination, leading to an extreme price fall. Y. Hashimoto et al. used actual foreign exchange market data and indicated that the market price is dependent on past price trends [5]. Thus, we think it is meaningful to focus on the phenomena brought about by investors' swarm behaviors. In particular, in the present study, we investigate two swarm behaviors: one that follows a trend in the historical price movement that is seen in general financial market and another that contradicts a trend in the historical price movement. We present a simple stochastic order-book model that captures these swarm behaviors of investors. From the price data derived from simulations using these models, we capture the behaviors of investors. We analyze the cumulative distribution of the price movement range when the price continuously move in a single direction.

2. Model

In this section, we describe the structure of our proposed model. Our model is applied to trading on exchanges; therefore, we begin by explaining the trading structure used by major exchanges. Major global financial exchanges use the continuous double auction mechanism. The key idea of this mechanism is the use of an electronic board called an order book where each trader writes bids and asks on and orders are matched on. In addition, two main order types are used by major exchanges. One is the limit order used to indicate a price upon which traders wish to have their orders executed, and another is the market order that is used without an indication of price. When sending an ask limit order, if there is a bid order on the order book that has the same or higher price than a trader's ask price, then the ask limit order will be matched with the bid order. Similarly, when sending a bid limit order, if there is an ask order on the order book that has the same or lower price than that of the bid order, then the bid limit order will be matched to the ask order. A market order is immediately matched with any existing order on the order book. When there are ask orders on the order book, any bid market order will be matched with the lowest priced ask order on the order book. Again, in the case when bid orders are on the order book, any incoming ask market order will be matched with the highest priced bid order on the order book. For matching the orders, the price priority rule is used. On the basis of the price priority rule, the priced highest bid order on the order book will be given priority over all other bid orders, and the lowest priced ask order on the order book will be given priority over all other ask orders. Another important rule is the time priority rule. If there are multiple orders on the order book with the same price, the oldest order is given priory against all other orders at that price, and will be executed first. Major exchanges around the world use both the price priority rule and the time priority rule. The trading price is the price of either the bid or ask order that was on the order book first.

We will next introduce several models that capture the continuous double auction mechanism. In this study, a stochastic model that uses the continuous double auction mechanism to capture the trading process is called the "stochastic order-book model". One of the pioneers of the "stochastic order-book model" is the model introduced by S. Maslov [6]. In this early model, limit order and market order were chosen with equal probability. Bid and ask orders were chosen with equal probability as well. The limit order price was selected by a uniform random number within a certain range from the last price. This is a very simple model, but it successfully captures the power law in the cumulative frequency distribution of the price difference gathered through simulations. Since this model, various stochastic order-book models have been proposed. For example, in Maslov's model, the order price is selected by a uniform random number; however, other models have looked at certain circumstances and used a selection trend or some distribution for selection of the order price [7–11]. These models can be broadly classified into two categories: one is the exclusion model where only one unit of order can be pooled in the same price, and another is the particle model where several orders can be pooled in the same price. In addition, Maskawa proposed a model where investors mimic the historical order trend [12]. In this model, with a certain probability, the priority given to an order's price is based on the price that has the greatest pooled order volume on the order book. Simulations