Physica A 420 (2015) 49–58

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Modeling sediment concentration in debris flow by Tsallis entropy

Vijay P. Singh [a,b,*], Huijuan Cui [c]

[a] Department of Biological & Agricultural Engineering, Texas A&M University, College Station, TX 77843-2117, USA
[b] Zachry Department of Civil Engineering, Texas A&M University, College Station, TX 77843-2117, USA
[c] Water management and Hydrological Science, Texas A&M University, College Station, TX 77843-2117, USA

HIGHLIGHTS

- Tsallis entropy is employed for modeling sediment concentration of debris flow.
- The Tsallis entropy-based debris flow concentration agrees with field data.
- The entropy-based model can compute the uncertainty associated with debris flow.

ARTICLE INFO

Article history:
Received 28 August 2014
Received in revised form 14 October 2014
Available online 5 November 2014

Keywords:
Hazard flow concentration
Entropy
Lagrange multipliers
Mountainous regions
Sediment concentration
Sediment particles

ABSTRACT

Debris flow is a natural hazard that occurs in landscapes having high slopes, such as mountainous areas. It can be so powerful that it destroys whatever comes in its way, that is, it can kill people and animals; decimate roads, bridges, railway tracks, homes and other property; and fill reservoirs. Owing to its frequent occurrence, it is receiving considerable attention these days. Of fundamental importance in debris flow modeling is the determination of concentration of debris (or sediment) in the flow. The usual approach to determining debris flow concentration is either empirical or hydraulic. Both approaches are deterministic and therefore say nothing about the uncertainty associated with the sediment concentration in the flow. This paper proposes to model debris flow concentration using the Tsallis entropy theory. Verification of the entropy-based distribution of debris flow concentration using the data and equations reported in the literature shows that the Tsallis entropy-proposed model is capable of mimicking the field observed concentration and has potential for practical application.

1. Introduction

A debris flow commonly comprises more than 50% sediment by volume and the sediment particles may range in size from clay to boulders several meters in diameter [1]. Thus, it is a dense, poorly sorted, solid–fluid mixture. In mountainous regions, prolonged heavy rainfall occurring over saturated hillslopes, earthquakes, and human activities are the main causes of debris flows. In debris flow, debris or rocks concentrate at the head of flow and move downslope with high concentration and strong destructive force. As they continue to flow downhill and through channels, with the addition of water, sand, mud,

* Corresponding author at: Department of Biological & Agricultural Engineering, Texas A&M University, College Station, TX 77843-2117, USA.
E-mail address: vsingh@tamu.edu (V.P. Singh).

http://dx.doi.org/10.1016/j.physa.2014.10.075

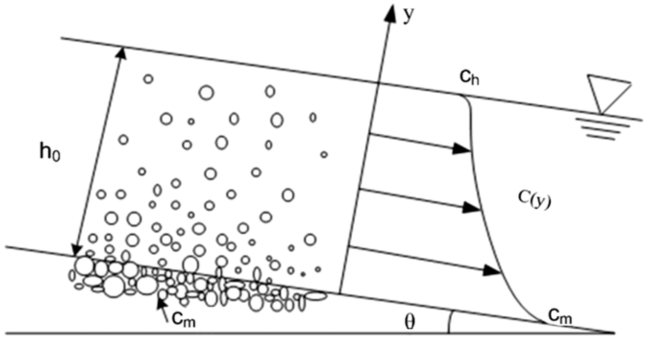

Fig. 1. Steady uniform debris flow.

trees, boulders, and other material they increase in volume and sediment concentration. Debris flows have a strong erosive force, grows during the movement of gathering debris eroded from the torrent bed or banks, and are capable of transporting huge volumes of sediment. Because of their power, these flows destroy whatever comes in their way, i.e., they kill people and animals; decimate roads, bridges, railway tracks, homes and other property; and fill reservoirs.

Debris flow is caused in three ways: (1) The gully bed material is mobilized or the sediment particles from the gully bed are entrained by water runoff; (2) a natural dam formed by a landslide fails; and (3) a landside block is liquefied. The discussion in this paper is restricted to the first way. Sediment concentration and sediment particle size are two fundamental factors that govern debris flow and its characteristics [2]. Of typical concern associated with debris flow are the formation, movement, and deposition of debris; delineation of flood zones; damage assessment; amongst others. To address these concerns, the equilibrium sediment concentration and its vertical distribution are needed. Debris flows, involving mixtures of debris and water, exhibit characteristics that are different from those of water flow or sediment-laden river flow. Although a physically-based hydraulic model can be constructed for modeling debris flow, uncertainties in debris flow variables and parameters of such a model may, however, limit its potential. Lien and Tsai [3] were probably the first to employ the Shannon [4] entropy for debris flow modeling. The objective of this paper is to employ the Tsallis [5] entropy for the determination of sediment concentration distribution in debris flow.

1.1. Notation and definition

Consider a debris flow over an erodible bed, as shown in Fig. 1. It is assumed that the flow is steady and uniform, where the depth of flow is h_0, and the sediment concentration decreases monotonically from a maximum value of c_m at the channel bottom to an arbitrary value of c_h at the water surface. Let $c(y)$ be the sediment concentration at a vertical distance y $(0 \le y \le h_0)$ from the channel bed. The sediment concentration is defined as the volume of sediment divided by the volume of fluid–sediment mixture and is dimensionless. Thus, it is expressed as a fraction or in percent by volume. For application of the Tsallis entropy, it is assumed that the time averaged sediment concentration C is a random variable.

2. Methodology for determination of debris flow concentration

Determination of debris flow concentration using the Tsallis entropy entails (1) definition of the Tsallis entropy, (2) specification of constraints, (3) maximization of entropy, (4) determination of the Lagrange multipliers, (5) determination of probability density function and maximum entropy, (6) cumulative probability distribution hypothesis, and (7) sediment concentration distribution. Each of these components is now discussed.

2.1. Definition of the Tsallis entropy

Let the concentration C in debris flow be the random variable with probability density function (PDF), $f(c)$. Then, the Tsallis entropy [5] of C, $H(C)$, can be expressed as

$$H(C) = \frac{1}{m-1}\left\{1 - \int_{c_h}^{c_m} [f(c)]^m \mathrm{d}c\right\} = \frac{1}{m-1}\int_{c_h}^{c_m} f(c)\left\{1 - [f(c)]^{m-1}\right\}\mathrm{d}c \qquad (1)$$

where c, $c_h \le c \le c_m$, is the value of random variable C, c_m is the maximum value of C or concentration at the bed, c_h is the concentration at the water surface, m is the entropy index, and H is the entropy of $f(c)$ or C. Eq. (1) is a measure of the uncertainty of variable C. The quantity $f(c)\mathrm{d}c$ defines the probability of sediment concentration occurring between c and $c+\mathrm{d}c$. The objective is to derive $f(c)$ which is accomplished by maximizing H, subject to specified constraints, in accordance with the principle of maximum entropy (POME) [6].