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The stationary state and gravitational temperature in a pure self-gravitating system

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HIGHLIGHTS

- A new concept of temperature called gravitational temperature is introduced.
- The concept of the gravitational thermal capacity is also introduced.
- The minimum mass is estimated for the pure self-gravitating system.
- The thermodynamic stability condition can be determined by the nonextensive parameter.

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ABSTRACT

The pure self-gravitating system in this paper refers to a multi-body gaseous system where the self-gravity plays a dominant role and the intermolecular interactions can be neglected. Therefore its total mass must be much more than a limit mass, the minimum mass of the system exhibiting long-range nature. The method to estimate the limit mass is then proposed. The nonequilibrium stationary state in the system is identical to the Tsallis equilibrium state, at which the Tsallis entropy approaches to its maximum. On basis of this idea, we introduce a new concept of the temperature whose expression includes the gravitational potential and therefore we call it gravitational temperature. Accordingly, the gravitational thermal capacity is also introduced and it can be used to verify the thermodynamic stability of the astrophysical systems.

1. Introduction

The self-gravitating system is a type of multi-body system naturally organized through the gravitational interactions. Planets like our Earth, stars like the Sun, and even the galaxies consisting of so many stars all belong to such systems. It is well known that in many cases the classical Boltzmann–Gibbs (BG) statistical method is not applicable to such astrophysical systems because the exponential distribution functions based on BG statistics would lead to many unreasonable theoretical results, such as the infinite mass and the negative capacity problem.

Since nonextensive statistics was proposed in 1988 [1], the theoretical successes in both astronomy and astrophysics [2–5] and space plasmas [6–11] have shown that the new statistical method may be suitable for describing many astrophysical systems. In this work, we apply the methods of nonextensive statistics to study the nonequilibrium property of a pure self-gravitating system. The pure self-gravitating system refers to a gaseous system in which no phase transition and no nuclear reactions take place and the self-gravity plays a dominant role.

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Obviously, such a pure self-gravitating system is a gaseous sphere consisting of the particles with self-gravitating interactions. In the classical BG statistics, the total energy of the gaseous sphere [12] is

$$E = -\frac{3}{2} N k_B \bar{T}, \tag{1}$$

and so the thermal capacity is

$$C_V = \frac{\mathrm{d}E}{\mathrm{d}\bar{T}} = -\frac{3}{2} N k_B, \tag{2}$$

where $\bar{T}$ is the average temperature of the system. Eq. (2) indicates that the system is thermodynamically unstable and will lead to the so-called gravothermal catastrophe, which was discussed in the system in a spherical adiabatic wall [13,14]. According to Eq. (2), the conclusion can be drawn that most self-gravitating systems do not exist in the universe for long-term, which, obviously, conflicts with the observation facts.

In the previous work, we divided the self-gravitating system as three kinds. In the first kind of system whose total mass is less than σM_e, where M_e is the mass of the earth and σ is a parameter whose value ranges about 10–300 [15], the intermolecular potential can play an important role. Under certain conditions, say, when the pressure is high enough near the center of the system, the phase transition related to the intermolecular interaction may take place, one result of which is the sign of the thermal capacity of the whole system to become positive, thus leading to thermodynamic stability of this kind of system. In the third kind of system whose total mass is more than $0.08M_{\odot}$, where $M_{\odot}$ is the solar mass, just as everyone knows, there exist nuclear reactions in the core. Obviously, the intermolecular interactions can be ignored due to very high temperature in such case. Therefore, the phase transition similar to that in the first kind of system does not appear. Although its heat capacity is negative, the energy compensation due to the nuclear reactions keeps thermodynamic stability of the whole system, which actually presents almost in all of the stars. In the second kind of system whose total mass is more than σM_e and less than $0.08M_{\odot}$, the short-range intermolecular potentials can also be neglected. It is clear that in such a system there is neither the phase transition related to the molecular potentials nor the nuclear reactions taking place. This kind of system is just the pure self-gravitating system we discussed in this paper.

The paper is organized as follows. The minimum mass as the pure self-gravitating system is studied in Section 2, the nonequilibrium stationary state and the Tsallis equilibrium is discussed in Section 3, the concept of gravitational temperature is introduced in Section 4, the thermodynamic stability condition (TSC) and the gravitational thermal capacity are studied in Section 5, and finally the conclusions and discussions are given in Section 6.

2. The minimum mass as the pure self-gravitating system

It is well-known that the intermolecular potentials have the nature of short-range interactions, then in the system consisting of many molecules, the total intermolecular potential is approximately proportional to the molecule number N. In contrast, the gravitational interactions have the nature of long-range, then the total gravitational potential is approximately proportional to some power of the molecule number N of the system, that is, $\sim N^n$, where n is a real number and $n > 5/3$ [15]. (This is a conservative estimate we give according to the constant density distribution. For a self-gravitating system with constant density, its total potential energy (absolute value) should be proportional to $N^{5/3}$. For a real self-gravitating system whose mass is more concentrated, the total potential energy is more than that of the system with constant density. This means the exponent may be $n > 5/3$.)

Obviously, with the increase of molecule number, the total gravitational potential increases more rapidly than the total molecular potential in the gaseous system. When the molecule number is large enough, the intermolecular potentials can be neglected compared with the gravitational potentials, thus the latter will play a dominant role in such system. Therefore, we now propose a method to estimate the minimum mass of such a pure self-gravitating system consisting of gaseous molecules.

For the laboratory system whose size is about dozens of meters, the self-gravitating potential energy can be ignored relatively to the interaction energy between the molecules. On the contrary, for the huge self-gravitating gaseous system, like the Sun, the molecular potentials are negligible relatively to the gravitational potential energy. Therefore, the minimum mass can be determined by just equating these two potential energies. For simplicity, we assume that the mass density of the pure self-gravitating system is a constant, and then we easily find the self-gravitating potential energy,

$$U = -\frac{6}{5}\frac{GM^2}{R}, \tag{3}$$

where M is the total mass of the system, and R is the radius of the gaseous sphere. Next, for calculating the interaction energy of molecules, we adopt the two-molecule interaction potential proposed by Lenard Jones [16],

$$\phi(r) = \phi_0 \left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6} \right], \tag{4}$$