Physica A 420 (2015) 324–330

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Entanglement witness for spin glass

C.Y. Koh [a,*], L.C. Kwek [a,b,c]

[a] National Institute of Education, Nanyang Technological University, 1 Nanyang Walk, Singapore 637616, Singapore
[b] Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore
[c] Institute of Advanced Studies (IAS), Nanyang Technological University, 60 Nanyang View, Singapore 639673, Singapore

ARTICLE INFO

Article history:
Received 24 May 2014
Received in revised form 14 September 2014
Available online 11 November 2014

Keywords:
Spin glass
Spin chain
Entanglement

ABSTRACT

We derived the entanglement witness (EW) in the Ising model for both the magnetic susceptibility and specific heat capacity of a spin glass. The magnetic susceptibility EW curve for $LiHo_xY_{1-x}F_4$ is formulated and compared with the existing data of $LiHo_{0.167}Y_{0.833}F_4$ to identify the entangled and unentangled regions. The EW for the magnetic susceptibility was found to cut the cusp of the experimental result at a critical temperature of about 0.2669 K. The specific heat capacity EW curve for Cu_xMn is formulated and compared with the existing data of $Cu_{0.279}$ Mn to identify the entangled and unentangled region for the various applied magnetic field *B*. With increasing *B*, the critical temperature where the EW curve intersects the experimental data, increases as well.

1. Introduction

One of the long standing open question in the condensed matter physics is whether a spin glass has a new phase transition at certain critical temperature. Basically, a spin glass consists of anti-ferromagnetic and ferromagnetic spins which are randomly distributed in a non-magnetic material such as copper and gold. Due to the random positions of the spins, the alignment of the spins will tend to align in the configured energy of the ferromagnet or anti-ferromagnet depending on the neighboring spins. It is because of this nature of random positioning and usually at unequal distance apart that the frustration in the interaction occurs. These two features – disorder and frustration – form the very foundation of what is termed spin glass [1–7]. It is well known that liquid and gas contain atoms or molecules that move randomly and without any order in space. It is precisely because of this that statistical mechanics is able to describe the properties of liquid and gas theoretically with statistics and probability due to their symmetry nature. In contrast, spin glass does not contain atoms that move around in the alloy randomly. As a matter of fact, it is believed to be quenched or fixed in position over a time scale greater than the age of the universe. As a result, symmetry is broken and the knowledge of statistical mechanics is not able to fully describe the physics of a spin glass.

Experimental studies have shown that a typical spin glass exhibits a cusp in the magnetic susceptibility at certain critical temperature for a low applied magnetic field. As most of these alloys contain a few percent of a magnetic element that is randomly distributed in the non-magnetic host, the critical temperature at which this cusp appears varies according to the level of impurity concentration [8]. Examples of such diluted alloys are copper and manganese, Cu_{1-x} Mn_x [9] or gold and iron, Au_{1-x} Fe_x [10]. Other alloys with insulation and conduction properties which found to be spin glass are europium

* Corresponding author.
E-mail address: marvelnature@gmail.com (C.Y. Koh).

http://dx.doi.org/10.1016/j.physa.2014.11.004
0378-4371/

strontium sulfur $Eu_x\ Sr_{1-x}\ S$ [11] and lanthanum gadolinium aluminum $La_{1-x}\ Gd_x\ Al_2$ [12]. In general, all thermodynamic functions should in some way behave singularly when a phase transition has occurred [7,13]. Therefore, the cusp in the magnetic susceptibility may suggest the occurrence of a phase transition. The effect of the sharp cusp becomes a broad maxima when a magnetic field of about 100 G is applied [10,14,15]. Besides being field dependent, some spin glasses are found to be frequency dependent [9,16]. In contrast to the cusp in the susceptibility, the specific heat capacities of $Au_{0.92}Fe_{0.08}$ [17] and CuMn [18] do not exhibit any sharp transition or singularity. Only a broad, smooth and rounded maximum is observed. In addition, the rounded maximum does not match the transition temperature of the magnetic susceptibility.

Based on the experimental results, models like the Edwards–Anderson (EA) [19] and Sherrington–Kirkpatrick (SK) [20] have been formulated in an attempt to understand the physics behind spin glass. The coupling in the EA model uses a random set of bonds that is usually taken from a Gaussian distribution. The random couplings represent site disorder and random Ruderman–Kittel–Kasuya–Yosida (RKKY) couplings [21–23]. Moreover, an asymmetric cusp is produced for both the magnetic susceptibility and specific heat capacity. Results by Fischer [24] have shown that the theoretical specific heat does not always fit the experimental one. It is only true for low temperature linear dependence for spin $S = \frac{1}{2}$. In a bizarre manner, the SK model which manages to produce a cusp in the magnetic susceptibility and specific heat capacity produced unphysical negative entropy. Due to the instability of the SK solution, Almeida and Thouless (AT) [25] divide the phase diagram for a spin glass into stable and unstable regions a line which is later named as the AT line. The instability of the solution was later found due to the treatment of all the replicas as indistinguishable. The unphysical negative entropy was later removed with the use of a replica symmetry breaking (RSB) scheme by Parisi [26–30]. Even then the method was found to be marginally stable. Despite the fact that different models and theories have been used to understand the physical nature of spin glass, there still remain many unaccounted experimental results which require a better theory to explain it. As all these theories have treated the spin glass in a classical sense, the quantization of the spins of the impurities is not taken into account [16]. However, the mathematical tools and new insights obtained in the study of spin glass were found to be useful in other areas of complex optimization problems [31], biological problems [32] and condensed matter [6,5,33]. In recent years, a quantum spin glass model of the form $LiHo_xY_{1-x}F_4$ has been extensively studied both experimentally and numerically [34–39]. The magnetic Ho^{3+} ions in these materials behave like effective Ising spins while the yttrium Y^{3+} are non-magnetic ions. With an x concentration of ≤ 0.25, a spin glass phase is found to exist. Nevertheless, it still remains as an open question of whether a spin glass or an antiglass spin phase exists at lower concentration.

In the early development of quantum mechanics, the Einstein, Podolsky and Rosen [40] paradox has given rise to the notion of nonlocal realistic description of nature, bringing the idea of entanglement to the forefront. Recently, entanglement has been identified as a valuable resource for quantum information processing and it is used extensively in the study of phase transition for condensed matter physics. Moreover, the study of entanglement in both quantum information science and condensed matter physics forms an interesting connection between them [41–47]. Since entanglement described the quantum correlations in a many-body system, one raises the question of whether entanglement can be quantified at the macroscopic level [48,49]. Indeed, many studies have been carried out in identifying the various experimental measurements used for the detection of entanglement in the macroscopic system. This quantity is called entanglement witness (EW). Studies show that some thermodynamical properties like magnetic susceptibility, heat capacity and internal energy can be used as EW to detect entanglement between the individual particles of a solid [50–52]. The advantage of using such EW is that the measurements are applied at the macroscopic level. Some recent studies have used magnetic susceptibility and heat capacity as EW to quantify between the entangled and non-entangled regions by matching the experimental results of materials with the theoretical EW [50,51,53–58]. Even though such studies have matched the EW successfully with the materials studied, little work has been carried out for spin glass. Moreover, the study of entanglement witness in spin glass is important due to the following reasons: (i) to find out whether entanglement can be used as an order parameter to describe the quantum phase transition; (ii) to determine the limits of entanglement in terms of size for a spin glass; (iii) to find out if entanglement is a robust measurement in the presence of temperature. With this motivation, we attempt to use the EW for magnetic susceptibility and specific heat capacity in quantifying the entangled region from the non-entangled one for some known experimental results of the glassy materials.

The paper is organized as follows. We begin in Section 2.1 by deriving the EW for magnetic susceptibility. The experimental results of the glassy materials are then plotted with the theoretical EW for the magnetic susceptibility. These results are presented and discussed in Section 3.1. The same is carried out for the EW of the specific heat capacity in Section 2.2 and the results discussed in Section 3.2. In Section 4, we summarize our results and indicate some possible future directions.

2. Theoretical formulation of entanglement witness

2.1. Magnetic susceptibility

In this subsection, we follow the work of Wieśniak et al. [50] in deriving the magnetic susceptibility as a macroscopic entanglement witness. We briefly mentioned the derivation for the magnetic susceptibility here as the full work can be found in Ref. [50]. For an arbitrary state of spin S particle, one has

$$\langle (S_x)^2\rangle + \langle (S_y)^2\rangle + \langle (S_z)^2\rangle = S(S+1) \tag{1}$$