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Taylor–Couette flow and a molecule dependent transport equation

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HIGHLIGHTS

- We apply a modified Navier–Stokes Equation to the Taylor–Couette flow.
- We report the first analytic solutions for velocity profiles in turbulent regime.
- Profiles are compared with the reported first direct numerical simulation results.

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ABSTRACT

We apply a previously derived and utilized a modified Navier–Stokes equation to the Taylor–Couette flow, that is fluid flow enclosed between two concentric cylinders where the inner cylinder is rotating with some constant speed and the outer cylinder is stationary or vice versa. We report the first analytic solutions describing velocity profiles of such a flow in a turbulent regime. The analytic profiles are compared with results of the reported first direct numerical simulation of Taylor–Couette flow in turbulent regime Pirro and Quadrio (2008).

1. Introduction

The Taylor–Couette flow, a complex and yet unresolved problem especially in terms of instability or turbulence as well as transition to turbulence, has attracted the attention of many distinguished scientists such as Newton, Stokes and Chandrasekhar. According to modern experimental observations there is just one direct transition from laminar to turbulent regime in the situation when the outer cylinder is rotating while there are at least two transitions, laminar to Taylor vortex flow, wavy vortex and then transition to turbulence while the inner cylinder is rotating.

In the paper [1] a new fluid equation modifying the Navier–Stokes equation was derived from the Boltzmann kinetic equation incorporating inelastic interactions. The first computational tests of this fluid equation applied to circular pipe and flat-plate systems were reported in Ref. [2] and specifically velocity profiles for the turbulent flow along a single wall were tested against experimental data and compared with standard log law of the wall. The result was consistent with experimental data. Previously large discrepancies of this log law with experimental data were reported in Ref. [3].

In this paper the new fluid equation is applied to the Taylor–Couette flow. The analytic solutions for the laminar and turbulent regime are presented and resulting velocity profiles are plotted and compared with a recent numerical simulation of Taylor–Couette flow in Ref. [4] by using two adjustable parameters appearing in the fluid equation.

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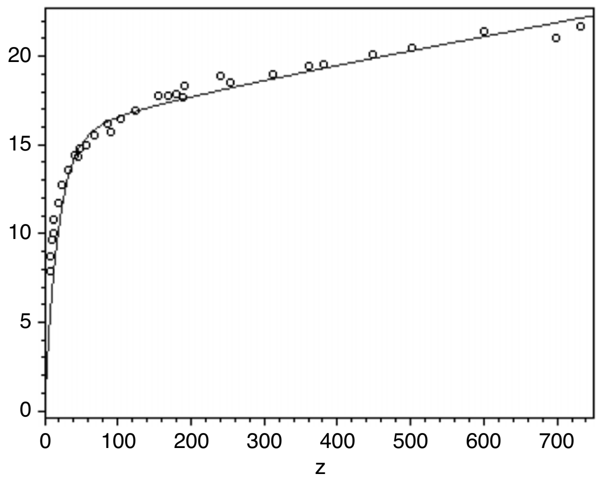

Fig. 1. Velocity profile $v(z)$ of turbulent flow along a single wall [2]. Solid line represents theoretical curve and circles represent experimental data from Ref. [8].

Since the technical aspects in the derivation of the new fluid equation are already presented in Ref. [1] we just mention here the very fundament, that is, a hypothesis that turbulence is a result of inelastic interactions of a quantum origin between particles of the fluid. For example rotational levels of molecules are excited due to collisions if the molecules have sufficient energy. Inelastic interactions between single atoms could be elucidated by quantum confinement [5]. Inelastic interactions are the source of deterministic chaos [6] that we associate with turbulence. The justification for the basic hypothesis that atoms and molecules are important when dealing with turbulence has been just summarized in a recent book [7]. Briefly put, experiments on turbulence have not been very accurate. However, the quantum kinetic theory depends on more accurate experiments using new apparatus as described in Ref. [7]. Further experiments are necessary such as those utilized in vacuum science and infrared spectroscopy from radiating turbulent gases (in an excited state). The quantum kinetic model of turbulence also tells us why turbulence is a highly energy dissipative process.

2. The new fluid equation as applied to Taylor–Couette flow

The fluid equation postulated in Ref. [1] is applied to Taylor–Couette flow, that is, the flow enclosed between two concentric cylinders, the inner cylinder of radius r_1 rotating at constant speed v and the outer cylinder of radius r_2 is stationary. The fluid equation for the mean velocity of the flow $U = (U_1, U_2, U_3)$ can be expressed in component form in the following way

$$\frac{\partial U_i}{\partial t} + U_j \partial_j U_i = -\frac{1}{\rho}\partial_j P_{ij} - \sigma\left(U_i + \frac{\Pi}{m}\right); \quad i, j = 1, 2, 3. \tag{1}$$

The last term in Eq. (1) is new, and is interpreted as a 'forcing' by a paddle wheel that increases the velocity of a particle in the fluid, σ is the probability per unit time that the particle of mass m is imparted with a constant momentum kick Π, and ρ is the density [1,2]. For the system under study, a rotating cylinder can be considered as a paddle wheel since a particle of mass m colliding with the rotating wall gains momentum Π. This is the manner of injecting energy into the system. On the other hand, fast particles in the vicinity of the wall also have radial velocities and collide with slower particles farther from the wall. The faster the particles near the wall, the greater is the probability of inelastic collisions of quantum origin, resulting in a loss or dissipation of momentum and energy.

Using a decomposition of the pressure tensor into its diagonal and off-diagonal parts, $P_{ij} = p\delta_{ij} + \sigma_{ij}$, where $\sigma_{ij} = \nu\left(\partial_i U_j - \partial_j U_i\right)$ is the shear stress tensor, p is the pressure, ν is the kinematic viscosity, and adopting incompressibility condition $\partial_j U_j = 0$, Eq. (1) reduces to

$$\frac{\partial U_i}{\partial t} + U_j \partial_j U_i + \sigma U_i = -\frac{\sigma \Pi}{m} - \frac{1}{\rho}(\partial_i p - \nu \partial_j^2 U_i); \quad i, j = 1, 2, 3. \tag{2}$$

The modified Navier–Stokes equation, Eq. (2), has been applied earlier to the case of turbulent flow along a single wall. A very good agreement of an obtained analytic exponential velocity profile [2] with experimental data in Ref. [8] is depicted in Fig. 1.

In this paper for Taylor–Couette flow, Eq. (2) is transformed to cylindrical coordinates

$$\frac{\rho}{\nu} r \frac{\partial U}{\partial t} + krU - r\frac{\partial^2 U}{\partial r^2} - \frac{\partial U}{\partial r} + \frac{U}{r} = gr \tag{3}$$

where the parameter $k = \frac{\rho\sigma}{\nu}$ is proportional to the probability of kicking particles to a different momentum, parameter $g = -\frac{k\Pi}{m}$, $U = U(r, t)$ is the mean tangential velocity and the pressure p is assumed to be constant in the tangential direction.