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Mean-field theory of modified voter model for opinions

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HIGHLIGHTS

- Mean-field theory gives analytical results for any transition matrix.
- Initial configurations are irrelevant.
- Noise and pressure determine both central plateau and boundary layer.
- Internal noise and external pressure scale with lattice size.

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ABSTRACT

We present a mean-field theory for the modified three-state voter model. We obtain analytical results in asymptotic states for a general transition matrix. Numerical simulations can be well described within the proposed formulations. Distributions for both central plateau and boundary layer can be written explicitly. The central plateau is determined solely by the ratio between the internal noise and the external pressure. In the bulk, these two factors substantially compete with each other. Near the boundary regions, both the factors work together to introduce the exponential distribution of transit layer. With the scaling, simulations on a large lattice can be mapped into a small lattice. The required computer time is hence reduced significantly.

1. Introduction

Opinion dynamics studies the evolution of opinions in a society [1–3]. Basically the opinions change. As time passes, some opinions can be going extinct. Different opinions can merge into a community consensus. Others may flip–flop from time to time. With the advances of socio-physics [4,5], the realm of classical statistical physics has been extended to cover social phenomena, such as exploring the emergence of unanimity among diverse opinions [6,7]. Researchers tend to capture the essence of such complicated evolutions, propose minimal models, and analyze possible correlations in the model with mathematical rigor. Theories of statistical physics can often provide a scientific rationale for reinterpreting or modifying the model to suit the purposes. One of the examples is the spin model on lattice [8–10]. On each site of the lattice, the corresponding opinion can be analogized as the different orientation of the spin. In physics model, the spins influence each other via spin–spin interactions, external magnetic fields, and thermal agitation. In opinion dynamics, each individual opinion can also be intervened via local influence and global influence [11]. The local influence arises from the opinions of neighbors, which vary from site to site; while the global influence results from the environment, which extensively acts on all sites.

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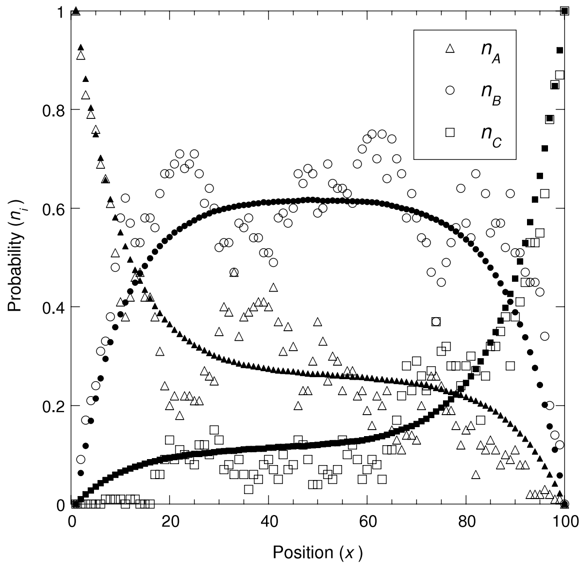

Fig. 1. Typical results of the model on a 100×100 lattice. The open symbols plot the data at $t = 10^6$; the filled symbols plot the temporal average from $t = 5 \times 10^5$ to 10^6. Parameters are $t_{AB} = 1 - t_{AC} = \frac{3}{4}$, $t_{BC} = 1 - t_{BA} = \frac{1}{4}$, $t_{CA} = 1 - t_{CB} = \frac{1}{2}$, and $q = p = 0.001$.

Along this line, the voter model can be a basic model for opinion dynamics in nature. Two opinions supposedly compete with each other via local influence [12,13]. More recently, a modified voter model was proposed to study the evolution of language decision in a newly unified state [14,15]. A certain update rules like global influence were included. Preliminary results of numerical simulations were reported. Boundary effects were found to be crucial. In this work, we present a mean-field approximation to the modified three-state voter model. We show that numerical results with the time average properties can be analytically described, including the boundary effects. The model is briefly reviewed in Section 2. The mean-field theory is proposed in Section 3. Discussions are presented at the end of this study.

2. Model

The model claimed to describe the competition between the primary languages used in Southeast Europe [14]. In a geostructure of regular networks, boundary effects are emphasized. The model addresses the opinion dynamics in general. The cellular automaton model is defined on a regular $L_1 \times L_2$ lattice. The lattice sites are labeled by (x, y), where $1 \leq x \leq L_1$ and $1 \leq y \leq L_2$. Each lattice site adopts one of the three opinions (or languages) denoted by A, B, and C. The model assumes a very strong influence due to neighbors. At each time step, the opinion at each site is switched to the one adopted by a randomly selected nearest-neighbor. The model also includes the internal noise q and the external pressure p. With a small probability q, at each time step, the opinion of each site changes spontaneously as prescribed by the following transition matrix,

$$\begin{pmatrix} 0 & t_{AB} & t_{AC} \\ t_{BA} & 0 & t_{BC} \\ t_{CA} & t_{CB} & 0 \end{pmatrix}, \tag{1}$$

where t_{ij} denotes the probability to change from i to j. As $t_{ii} = 0$, the three constraints can be written as

$$\sum_j t_{ij} = 1, \tag{2}$$

where $i, j = A, B, C$.

With a small probability p, at each time step, the external pressure forces opinions A and C to change into B. The model assumes the boundary conditions at two fixed ends ($x = 1$ and $x = L_1$). One side of the boundary is fixed to A and the other to C. Basically the dynamics in bulk favors the emergence of B; while the boundary condition forces A and C to become dominant near the two boundaries. Opinion B is expected to survive only in the middle region of the lattice. To further assist the survival of B, the initial configuration is setting to have A and C dominant near two different boundaries, and B dominant in the middle. Typical results of numerical simulations are shown in Fig. 1. Since the distribution is homogeneous in y direction, we plot the x dependence for the probability of each opinion (n_i). The boundary at $x = 1$ is fixed to A; and $x = L_1$ is fixed to C. Opinion competition in reality corresponds to the result at a single time-point in simulation. One does not anticipate a smooth distribution even in the asymptotic states. Strong fluctuations are expected, as shown by the open