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A general methodology for population analysis

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HIGHLIGHTS

- We rely on fundamental concepts population information and population entropy.
- Population entropy is, by definition, uncertainty, related to the population.
- Utilization, information stiffness and size of the population are three key metrics.
- Information (entropy) consists of null, elastic (Hooke's) and synchronization parts.
- Information linear and (a)symmetrical populations are introduced and studied.

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ABSTRACT

For a given population with N — current and M — maximum number of entities, modeled by a Birth–Death Process (BDP) with size $M + 1$, we introduce utilization parameter ρ, ratio of the primary birth and death rates in that BDP, which, physically, determines (equilibrium) macrostates of the population, and information parameter ν, which has an interpretation as population information stiffness. The BDP, modeling the population, is in the state n, $n = 0, 1, \ldots, M$, if $N = n$. In presence of these two key metrics, applying continuity law, equilibrium balance equations concerning the probability distribution $p_n, n = 0, 1, \ldots, M$, of the quantity N, $p_n = Prob\{N = n\}$, in equilibrium, and conservation law, and relying on the fundamental concepts population information and population entropy, we develop a general methodology for population analysis; thereto, by definition, population entropy is uncertainty, related to the population. In this approach, what is its essential contribution, the population information consists of three basic parts: elastic (Hooke's) or absorption/emission part, synchronization or inelastic part and null part; the first two parts, which determine uniquely the null part (the null part connects them), are the two basic components of the Information Spectrum of the population. Population entropy, as mean value of population information, follows this division of the information. A given population can function in information elastic, antielastic and inelastic regime. In an information linear population, the synchronization part of the information and entropy is absent. The population size, $M + 1$, is the third key metric in this methodology. Namely, right supposing a population with infinite size, the most of the key quantities and results for populations with finite size, emerged in this methodology, vanish.

1. Introduction

1.1. In this work, relying on the fundamental concepts information i and entropy S of the given *population*, with N — current and M — maximum number of *entities*, modeled by a Birth–Death Process (BDP) with size $M + 1$, we develop a *general*

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methodology for population analysis; the BDP modeling (the behavior of) the population, is in the state n, $n = 0, 1, \ldots, M$, if $N = n$. Thereto, by definition, the *population entropy* $S = E(i)$ is the *uncertainty*, related to the population. Formally, we accept Gibbs formula for entropy and its adaptation to Random Variables, proposed and firstly applied by Shannon (but, only its discrete version).

Three key metrics are essential in the proposed population analysis. The first, *utilization parameter* ρ of the population, ratio of the primary birth and death rates in the BDP, modeling the population, physically, defines *(equilibrium) macrostates* of the population. The second, *information parameter* ν of the population, has a physical interpretation as *population information stiffness*. In presence of these two key metrics, by applying of the *continuity law*, equilibrium balance equations concerning the probability distribution p_n, $n = 0, 1, \ldots, M$, of the quantity N, $p_n = Prob\{N = n\}$, in equilibrium, *elastic* (*Hooke's*) or *absorption/emission*part, $i_{el} = i_{abs/emi}$, and *synchronization* or *inelastic* part, $i_{syn} = i_{inel}$, of the *population information* **i**, are obtained. The population information *i* has possible values i_n, $i_n = -\ln p_n$, $n = 0, 1, \ldots, M$; while p_n *is weight, i.e. probability mass, associated to the state n of the population*, then i_n *is quantity of information the population possesses in the state n*. The information parts i_{el} and i_{syn} form the *Information Spectrum* (*IS*) of the population (as its two basic components). Applying the conservation law, i_{el} and i_{syn} are connected by the null part, $i_{null} = i_0$, of the whole information *i*. Thus, the null part, i_{null}, and, consequently, the whole information *i*, are uniquely determined by the IS of the population, $\Delta i = i - i_{null}$. The basic parts, S_{null}, $S_{el} = S_{abs/emi}$ and $S_{syn} = S_{inel}$, of the population entropy $S = E(i)$, being mean values of the corresponding basic parts of the population information *i*, $S_{null} = i_{null}$, $S_{el} = E(i_{el})$ and $S_{syn} = E(i_{syn})$, appear as a consequence from this information division; the same is true for their explicit expressions. In the case of an *information linear population*, when, by definition, $i_{syn} \equiv 0$ (and, clearly, $S_{syn} \equiv 0$), population information *i* reduces to its *regular* part, $i = i_{reg}$, $i_{reg} = i_{null} + i_{el}$ (then, clearly, $S = S_{reg}$, $S_{reg} = S_{null} + S_{el}$). The most of the basic results in this work concern populations with finite size — the size $M + 1$ of the population is the third key metric in this methodology. Namely, right supposing a population with *infinite size* (the number N of entities is not limited), the most of the key quantities and results, emerged in this methodology, vanish. So, *half loading point*, the value $\rho_{M/2}$ of parameter ρ, at which the average number $\bar{N} = E(N)$ of entities in the population with size $M + 1$ equals $M/2$, is very important, and *maximum uncertainty point*, the value $\rho_{M,max}$ of parameter ρ, at which population entropy S has maximum, is of crucial significance. Nevertheless, in this methodology framework, we give the corresponding results, concerning populations with infinite size.

In our consciousness, the proposed methodology has interdisciplinary character and, consequently, can intervene in various areas. In that sense, our work does not pretend to argue about the place and meaning of population information and entropy. Nevertheless, if the principal standpoints about population information and entropy, necessary for developing of the proposed methodology, are, at least partially, close to the points of view about these concepts, presented in, say, some remarkable published works (we allow ourselves to refer to the book [1]), then we will be glad.

1.2. In Section 2, in presence of parameter ρ, after applying continuity and conservation laws, basic equations for the probability distribution $p_n(\rho)$, $n = 0, 1, \ldots, M$ of the quantity N, are obtained. In such a way, a family BDPs with size $M + 1$, related to a polynomial $f_M(\rho)$ of degree M with respect to parameter ρ, is obtained. This family BDPs models the *direct family of populations* concerning the quantity N and describing the given population. Then, any value $\tilde{\rho}$ of parameter ρ defines one (*equilibrium*) *macrostate* of the given population, where the quantity N follows probability distribution $p_n(\tilde{\rho})$, $n = 0, 1, \ldots, M$.

Simultaneously with the direct family of populations, for describing the given population, the *inverse family of populations* concerning the quantity ${}^{M}N$ is introduced. Here, by definition, ${}^{M}N = n$ if and only if $N = (M - n)$, $n = 0, 1, \ldots, M$ (the state n of the inverse population is the state $(M - n)$ of the direct population), and, clearly, we have the same balance equations for both, direct and inverse family of populations. In the state n of the inverse population, $({}^{M}i)_n(\rho) = i_{M-n}(\rho)$, and so, information $({}^{M}i)(\rho)$ (and, consequently, entropy $({}^{M}S)(\rho)$) of the inverse population is determined. From the obtained basic equations for $p_n(\rho)$, $n = 0, 1, \ldots, M$, a representation of the information $i(\rho)$, with *nominal part*, $i_{nom}(\rho)$, and *additional part*, $i_{add}(\rho)$, is obtained (clearly, this is followed by a representation of the entropy $S(\rho)$, with *nominal part*, $S_{nom}(\rho)$, and *additional part*, $S_{add}(\rho)$). Then, by rearrangement of this population information representation, the basic parts $i_{null}(\rho)$, $i_{el}(\rho)$ and i_{syn} (i.e. $({}^{M}i)_{null}(\rho)$, $({}^{M}i)_{el}(\rho)$ and $({}^{M}i)_{syn}$) of information $i(\rho)$ (i.e. $({}^{M}i)(\rho)$), are obtained. Now, information parameter is introduced: a value ν, $\nu > 0$ of the information parameter of the given population, described by the direct and inverse family of populations, is related to every pair of values of parameter ρ, ρ' and ρ'', $\rho' < \rho''$, inverse symmetrical with respect to the point $\rho_{M,max}$ ($\rho' \cdot \rho'' = \rho^2_{M,max}$), $\rho^{'/''} = \rho_{M,max} \cdot \exp(\mp\nu)$. Then $i_{el}(\rho') = \nu \cdot N$, $({}^{M}i)_{el}(\rho'') = \nu \cdot ({}^{M}N)$, and parameter ν has interpretation as population information stiffness. Even more, what is highly significant, it gets another, deeper (physical) interpretation as *primary information quantum of absorption/emission*: by transition $n \to n+1$, $i_{el}(\rho')$ increases by amount ν – for $\rho' \in (0; \rho_{M,max})$ the given population functions in *information elastic regime*, and $({}^{M}i)_{el}(\rho'')$ decreases by amount ν – for $\rho'' \in (\rho_{M,max}; +\infty)$ the given population functions in *information antielastic regime*. If $\rho' = \rho'' = \rho_{M,max}$, then $\nu = 0$, and by transition $n \to (n \pm 1)$ the population neither absorbs, nor emits (a quantum of) information; the change of information is due to the different values of i_{syn} in the states n and $(n \pm 1)$ — the population functions in *information inelastic regime*.

In the proposed Methodology for population analysis, the quantity N (i.e. ${}^{M}N$) senses (i.e. reflects) the external influences on the population through the relative magnitude and the degree of (mutual) connectivity among the basic parts of the population information $i(\rho)$ (i.e. $({}^{M}i)(\rho)$) and of the population entropy $S(\rho)$ (i.e. $({}^{M}S)(\rho)$), as well as through the population regime of work together with its change, and, in this sense, the methodology is an unified approach. In this direction, in Section 3, a few interventions of the methodology are shortly described; especially, a possibility for *rejuvenation* of the