Physica A 416 (2014) 361–370

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Evolution of altruism in spatial prisoner's dilemma: Intra- and inter-cellular interactions

Hiroki Yokoi [a], Takashi Uehara [a], Tomoyuki Sakata [b], Hiromi Naito [c], Satoru Morita [c], Kei-ichi Tainaka [a,*]

[a] *Graduate School of Science and Technology, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432-8561, Japan*

[b] *Center for Information Infrastructure, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432-8561, Japan*

[c] *Department of Mathematical and Systems Engineering, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432-8561, Japan*

HIGHLIGHTS

- Iterated prisoner's dilemma game is studied on lattices with colony structure.
- Simulations are carried out among four typical strategies.
- Various dynamics emerge, and every strategy can be a winner.
- All Cooperation (AC) can win, when colony size or noise level increases.
- We analytically derive the condition for AC to win on one-dimensional lattice.

ARTICLE INFO

Article history:
Received 7 June 2014
Available online 10 September 2014

Keywords:
Iterated prisoner's dilemma
Intra-cellular interaction
Noise level
String patterns
Evolution of altruism

ABSTRACT

Iterated prisoner's dilemma game is carried out on lattice with "colony" structure. Each cell is regarded as a colony which contains plural players with an identical strategy. Both intra- and inter-cellular interactions are assumed. In the former a player plays with all other players in the same colony, while in the latter he plays with one player each from adjacent colonies. Spatial patterns among four typical strategies exhibit various dynamics and winners. Both theory and simulation reveal that All Cooperation (AC) wins, when the members of colony or the intensity of noise increases. This result explains the evolution of altruism in animal societies, even though errors easily occur in animal communications.

1. Introduction

Various forms of cooperation emerge in human and non-human societies without central authority. An iterated prisoner's dilemma (IPD) game clearly illustrates that cooperation can occur in situations where individuals (players) take care of themselves and their own first [1–3]. So far, many authors have reported optimal strategies against an egoist. Axelrod and Hamilton have reported that the strategy tit-for-tat (TFT) is a winner for computer game contests [1,4]. Several authors, however, have reported that Pavlov (PAV) outperformed TFT, when players contain noise (error) [5,6]. Both strategies TFT and PAV can explain the evolution of cooperation, especially on a lattice [3,7–10]. However, both TFT and PAV are far from a moral standard for a person to follow, since they are based on "revanchism": a player of TFT or PAV immediately betrays, if they are betrayed.

* Corresponding author. Tel.: +81 53 478 1228; fax: +81 53 478 1228.
E-mail address: kei_tainaka@yahoo.co.jp (K.-i. Tainaka).

http://dx.doi.org/10.1016/j.physa.2014.09.003

The IPD game is played by a pair of players. Per one game, N moves are repeated. In each move, both players take one of two options: either to cooperate or to defect. If both players cooperate, both get the pay-off R. If one cooperates and the other defects, then the former (latter) gets pay-off S (T). If both defect, both get pay-off P. In the present paper, we apply a standard model:

$$(T, R, P, S) = (5, 3, 1, 0). \tag{1}$$

In each move, a player always gets higher payoffs, when the player defects, or when the opponent player cooperates.

Perhaps, the most widely accepted altruism is the "Golden Rule": In the context of the IPD game, the Golden Rule would seem to imply that you should always cooperate. This interpretation suggests that altruism may be strategies close to All Cooperation (AC). Altruistic behaviors widely evolve in animal societies [11,12]. Nevertheless, AC has been considered to be inferior to other strategies, especially to All Defection (AD) and PAV. It is, therefore, difficult that AC wins the game [12,13].

In the present article, we apply a spatial prisoner's dilemma with error level (x) [2,14,15]. One of distinct features of our simulation is "intra-cellular interaction". Each cell is regarded as a "colony (family)" which contains $(m + 1)$ players with identical strategy. This idea comes from the fact that big colony species, such as ants or wasps, evolve the altruism. For simplicity, the system contains only four strategies TFT, AD, PAV and AC. We mainly report the results on one-dimensional (1-d) lattice. In the case of 2-d lattice, we fix at $x = 0.1$.

Simulations on 1-d lattice reveal the single winner; in most cases, only one strategy can occupy all cells in equilibrium. When $m = 0$, either AD or TFT tends to be a winner. If m slightly increases, TFT or PAV wins. For a large value of m, only AC wins. At the boundary between different winners, various "string patterns" are observed. Theory also proves that AC can be a winner, when m or x increases. The results on 2-d lattice are similar to those on 1-d lattice. However, string patterns never emerge on 2-d lattice. At the boundary between different winners, coexisting patterns of plural strategies are observed.

2. Model

Each lattice cell means a colony (family) which contains $(m + 1)$ players with identical strategy. Simulations are mainly carried out on 1-d lattice. Initial condition is set to be random, where four strategies TFT, AD, PAV and AC occupy with equal probability (1/4). To update the cells, we adopt the competition rule between two neighboring cells. We randomly choose neighboring colonies α and β. A player in colony α plays with intra- and inter-cellular opponents. In the former case, the opponents are m other players in the same colony. In the latter case, a player plays with one player each from adjacent colonies. Hence each player plays $m + z$ games, where z is the number of adjacent colonies. The fitness of colony α (or β) is defined by the total payoff over $m + z$ games. When the fitness of α is larger than that of β, then both colonies become the strategy of α (and vice versa). If both have the same fitness, both become either strategy of α or β with the probability 1/2.

3. Theory on average payoff

3.1. Game between the same strategy

Throughout our theory, we assume the infinity of moves ($N \to \infty$). If each colony has an infinity of players ($m \to \infty$), intra-colony interactions become dominant. In this case, we can prove that the winner is the strategy AC. We consider the IPD game between the identical strategy, and obtain average pay-off (A) of each player per one step (move).

(1) No memory strategies.

We consider the game of AC vs. AC. The strategy AC has no past memory, because it always takes the option C. In each move, an error occurs by the probability x, where $0 \le x \le 1/2$. Three cases may occur (i) both players have no error, (ii) only one player has error, and (iii) both players make errors. In cases (i)–(iii), the total pay-off ($2A$) of both players is given by 6, 5 and 2, respectively. Hence, we have

$$2A = 6(1 - x)^2 + 5 \times 2(1 - x)x + 2x^2.$$

It follows that

$$A = 3 - x - x^2. \tag{2}$$

Similarly, if both players are the strategy AD, we get

$$A = 1 + 3x - x^2. \tag{3}$$

(2) PAV vs. PAV.

Pavlov (PAV) shows the option C at the first move. If the opponent player shows D at move $n(1 \le n \le N)$, then PAV changes option at the next move [5]. We consider the case that both PAV players play the IPD game. At move n, both players show either option C or D. Let $P_{jk}(n)$ be the probability that a player takes the option j and the opponent player shows k ($j, k =$ C or D). The probability $P_{CC}(n + 1)$ at the next move is given by

$$P_{CC}(n + 1) = (1 - x)^2 P_{CC}(n) + x^2 P_{CD}(n) + x^2 P_{DC}(n) + (1 - x)^2 P_{DD}(n). \tag{4a}$$