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The first order phase transitions in the multisite spin-1/2 model on a pure Husimi lattice

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HIGHLIGHTS

- All possible physical phases of the model are found.
- The first order phase transitions of the model are found and discussed.
- The formation of the magnetization plateaus for low temperatures is shown.
- The full structure of the ground states of the model is found.
- The existence of the single-point ground states is demonstrated.

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ABSTRACT

We investigate the multisite spin-1/2 Ising-like model in the presence of the external magnetic field on the pure Husimi lattice with three sites in the elementary polygon ($p = 3$) and coordination number $z = 6$ which represents the simplest approximation of the model on the two-dimensional triangular lattice which takes into account its elementary geometric structure. It is shown that, depending on the values of the parameters of the model, at maximum three different phases exist in the model. The existence of the first order phase transitions between various phases is demonstrated and studied in detail. The total magnetization per site is analyzed as function of the temperature as well as of the external magnetic field. It is shown that for low temperatures the model exhibits a typical behavior of geometrically frustrated systems. All possible ground states of the model are found and discussed.

1. Introduction

During last few decades, a lot of classical Ising and Ising-like models have been investigated on various pure Husimi trees and lattices [1–3] (see the next section for their exact definition) which play important role in describing many interesting physical systems such as amorphous solids [4], spin liquids [5], the Ising spin glasses [6–9], various polymer models [10–15], abelian sandpiles [16,17], lattice gases [18], and ^{3}He systems [19]. At the same time, various models on pure Husimi trees and lattices are especially useful for investigation of physical systems in which multisite interactions play important role, i.e., for investigating such systems as, e.g., binary alloys, rare gases, lipid bilayers [20], or spin glasses [21]. Strictly speaking, there exist two main reasons for which various classical Ising and Ising-like models on pure Husimi trees and lattices are interesting from theoretical point of view (see, e.g. Refs. [22–34] as well as references cited therein). First of all, it is well-known that all such classical spin models on pure Husimi trees and lattices can be usually solved exactly at least in the closed

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form of recursion relations and, at the same time, these models simulate properties of real systems more appropriately than, e.g., the mean field approach [35–37].

Maybe the most interesting Husimi systems from phenomenological point of view are the pure Husimi trees and lattices which consist of elementary triangles and which have coordination numbers $z = 4$ and $z = 6$. The first one represents an appropriate approximation of real kagome lattice and the second one can be used for approximative description of the triangular lattice. They are especially interesting for investigating various antiferromagnetic systems which exhibit strong geometric frustration [38]. For example, quite recently it was shown that the antiferromagnetic Ising model in the presence of the external magnetic field as well as in the presence of the multisite interaction on the pure Husimi lattice built up from elementary triangles with the coordination number $z = 4$ is exactly solvable in fully analytical way [34,39]. On the other hand, however, it seems that the corresponding models on the pure Husimi lattice built up from elementary triangles but with coordination number $z = 6$ exhibit much more complicated behavior. In this respect, maybe the most peculiar behavior (such as the existence of bifurcations, the so-called n-cycle solutions, chaotic behavior, etc.) are reported in the papers in which the models with the presence of the multisite interaction on such kind of trees or lattices are investigated (see, e.g., Refs. [25,30,35,36] and references cited therein). In this respect, in the present paper we would like to return to the analysis of the pure multisite classical spin-1/2 model on this "triangular" pure Husimi lattice with coordination number $z = 6$ by using an exact numerical analysis of the full system of three recursion relations which takes into account basic triangular symmetry of the lattice (see the next section for details). It is shown that when entire symmetry of the model is taken into account, then, the model exhibits well defined physical solutions for arbitrary values of the parameters of the model. It also means that, in this case, none from above mentioned exotic behaviors of the model exist regardless of the parameters of the model. Besides, we shall show that, for given values of the parameters, the model can have at maximum three different physical phases which are related to the corresponding three independent stable fixed points of the system of recursion relations. On the other hand, the model also exhibits the first order phase transitions between various physical phases detailed analysis of which will be also performed. In addition, it is shown that for low enough temperatures the model shows a behavior which is typical for geometrically frustrated models. Namely, the model exhibits the existence of the magnetization plateaus as well as the single-point ground states with definite values of the magnetization, which are realized only for some exact values of the external magnetic field.

2. Model

We consider the classical multisite spin-1/2 model on the pure Husimi lattice with $p = 3$ and $q = z/2 = 3$ (see Fig. 1), where p denotes the number of sites in the single polygon and q is the number of polygons that meet at each site.[1] The Hamiltonian of the model is

$$\mathcal{H} = -J \sum_{\langle ijk \rangle} s_i s_j s_k - H \sum_i s_i, \tag{1}$$

where each variable s_i acquires one of the two possible values ± 1, J is the multisite interaction among spin variables within single polygons (triangles), and H is the external magnetic field. In Eq. (1), the first sum runs over all triangles and the second sum runs over all sites.[2]

Hence, the partition function of the model given by Hamiltonian (1) is

$$Z \equiv \sum_s \exp\left(-\beta \mathcal{H}\right) = \sum_s \exp\left(K \sum_{\langle i\,j\,k \rangle} s_i s_j s_k + h \sum_i s_i\right), \tag{2}$$

where $\beta = 1/(k_B T)$, T is the temperature, k_B is the Boltzmann constant, $K = \beta J$, and $h = \beta H$. Standardly, the sum over s in Eq. (2) means the summation over all possible spin configurations on the lattice.

It is well known that each classical statistical model on arbitrary recursive lattice can be investigated numerically by using the method of recursion relations (see, e.g., Ref. [41]). The model defined by the Hamiltonian (1) is not an exception to this rule. However, the corresponding recursion relations must be constructed in the form which takes into account all symmetries of the problem. In this respect, although, at the first sight, it seems that it is enough to work with single recursion relation in investigating the model with pure multisite interaction on the pure Husimi lattice shown in Fig. 1 (as it was always supposed in earlier papers, see, e.g., Refs. [25,30,35,36]), i.e., that it is not necessary to divide the lattice into the appropriate number of sublattices, nevertheless, as we shall see in the present paper, for a complete treatment of the problem it is absolutely necessary to take into account the triangular symmetry of the model.[3] To this end, it is quite

1 For completeness, let us remind the general definition of a pure Husimi lattice used in this paper. A *Husimi tree* is a connected graph in which no line lies on more than one cycle. On the other hand, a *pure Husimi tree* is a special type of Husimi tree which consists of only one type of figure (lines, triangles, etc.) out of which it is built up. Then, the pure Husimi lattice is defined as the interior of the corresponding pure Husimi tree, i.e., we suppose that we are located deep inside of the graph, where all sites are equivalent to each other.

2 Let us also remark that the pure multisite model in the zero external magnetic field on the triangular lattice was introduced in Ref. [40] by Baxter and Wu and therefore the model is usually called the Baxter–Wu model.

3 Strictly speaking, from pure mathematical point of view, the dynamical system described by the single recursion relation has the right to exist. However, it does not describe the multisite model neither on the studied pure Husimi tree nor on the corresponding pure Husimi lattice.