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Analysis of a decision model in the context of equilibrium pricing and order book pricing

D.C. Wagner *, T.A. Schmitt, R. Schäfer, T. Guhr, D.E. Wolf

Faculty of Physics, University of Duisburg–Essen, Lotharstrasse 1, 47057 Duisburg, Germany

HIGHLIGHTS

- Agent-based models help achieving microscopic understanding of financial markets.
- Two aspects to consider: decision making and price formation.
- Simple decision model in two pricing schemes: supply–demand balance and order book.
- Interesting features in order book setting, not present in equilibrium pricing.
- Nontrivial behavior of order book volumes which reminds of trend switching.

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ABSTRACT

An agent-based model for financial markets has to incorporate two aspects: decision making and price formation. We introduce a simple decision model and consider its implications in two different pricing schemes. First, we study its parameter dependence within a supply–demand balance setting. We find realistic behavior in a wide parameter range. Second, we embed our decision model in an order book setting. Here, we observe interesting features which are not present in the equilibrium pricing scheme. In particular, we find a nontrivial behavior of the order book volumes which reminds of a trend switching phenomenon. Thus, the decision making model alone does not realistically represent the trading and the stylized facts. The order book mechanism is crucial.

1. Introduction

A common method in the economic literature to determine the price of an asset is the concept of equilibrium pricing [1,2]. The price is the result of the available supply and demand. In the context of stock markets supply and demand are often identified with the market expectation of the traders to sell or buy a stock. For example, such a pricing scheme is frequently employed within Ising-type models [3–5], where the market expectation of each trader is symbolized by a spin on a lattice. There the decision to sell or buy a stock is mapped to spin down and up. In each time step an update of the market expectations is performed by taking the market expectations of the nearest neighbors into account [3]. A possible criterion to derive the equilibrium price is to take the average of all market expectations.

In contrast, the standard pricing mechanism at stock exchanges is the double auction order book [6,7]. The order book lists all current propositions to sell or buy at a given price. This information is available to all traders. Here, the price is the result of a new incoming order matching a limit order already in the book.

* Corresponding author. Tel.: +49 203 379 4737.
E-mail address: daniel.wagner@uni-due.de (D.C. Wagner).

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The empirical return time series show a collection of remarkable properties, which are called stylized facts; see Ref. [8] for a review. The empirical distribution of returns has heavy tails, *i.e.*, the tails are more pronounced compared to a normal distribution [9–12]. In the literature many approaches are discussed to explain this feature [13–21]. One point of view claims that large trading volumes are responsible for the heavy tails [13–18]. Another approach holds gaps in the order book structure responsible. If gaps are present between the price levels occupied by limit orders, even small volumes can cause large price shifts [19]. According to this reasoning the order book plays an important role in the emergence of heavy tails.

Agent-based modeling allows us to microscopically understand trading mechanisms [22–30] and to study trading strategies [31,32]. There is ample literature on agent-based models for financial markets. The approaches differ in the decision model, in the pricing scheme and in the analyzed observables. For instance, Alfi et al. [33,34] are able to reproduce some stylized facts of financial time series. They employ fundamentalist agents which tend to stabilize the market and chartist agents which induce destabilization. Their price formation is inspired by Walras' equilibrium price adjustment mechanism [35]. Giardina et al. [36] have a look at the microscopic origin of volatility clustering with the help of an extended minority game in which an agent's market expectation to buy or to sell an asset is represented by a binary variable. The price develops proportional to the sum of all market expectations. As a result they arrive at an interpretation for long-range correlation effects on the volatility of financial markets. Preis et al. [37] use an order book pricing with zero-intelligence traders that submit market and limit orders. In their model, they induce local trends by assuming an asymmetric probability to buy or sell, respectively. Heavy-tailed return distributions are achieved when coupling the order entry depth to the prevailing trend.

Here, we use the agent-based stock market introduced by Schmitt et al. [38] as a basis. It implements a double auction order book. The model is capable of reproducing the gap structure described in Ref. [19] and yields heavy-tailed return distributions. While Schmitt et al. employ traders that act randomly, independently of each other and follow no strategy, we incorporate this model by designing a trader that acts with respect to a decision model. Our traders are indirectly coupled to each other because their decisions influence the price which, in turn, affects their decision to buy or to sell an asset; see Section 2. Since the model has two parameters we initially analyze the parameters' influence on the price with an equilibrium pricing. Then we analyze the persistence of this behavior if we put the traders' decision model into an order book setting. As results of our simulations we discuss the return distribution and the order book structure in Section 3. We conclude our results in Section 4.

2. Model

We present a decision model in an equilibrium pricing and discuss how to implement it in an order book driven agent-based model. In reality, traders employ all kinds of strategies. Unfortunately, it is not possible to capture the decision making process of any trader. Furthermore, his strategy could be erratic or could at least contain erratic elements, so that it would be impossible to exactly determine how he would react in a particular case. Also, the more complex a system is, the less it is possible to trace back features in the observables, *e.g.* the price, to individual decisions. Therefore we restrict ourselves to a simple decision model where this is still possible.

In an agent-based model for a financial market the decisions of every trader (agent) and their influence on the price are simulated. Thus, agent-based models have to take into account two different aspects: decision making and price formation. A decision model describes how a trader reacts on events, *e.g.* when the price changes, and price formation is a mechanism describing how decisions manifest themselves in the price. Here, we analyze one decision model in combination with two different pricing models.

Our decision model demands that every trader has his own price estimation for a traded asset. Of course this influences his market expectation and, thus, his decision either to buy or sell. Depending on the current asset price his individual price estimation develops in time. He changes his decision to buy or to sell, respectively, depending on the relative deviation between the individual and the asset price; a distribution function determines the probability that he changes his market expectation.

The first pricing model calculates the relative price change between two time steps as the mean value of the market expectations of all traders. This corresponds to an equilibrium pricing, balancing supply and demand. In the second price formation scheme the price is determined in the framework of an order book setting. Here, we embed the decision model into an agent-based model with a double auction order book pricing. We will now look deeper into these aspects.

2.1. Decision making

By their very nature decision making models are capable of describing a large variety of scenarios, wherever a choice between alternatives has to be made. Contrary to approaches in the literature where the decisions are based on mutual decisions between the agents [39], the decisions in the present setting are only made with the reference to a general trend, which can be understood as a mean field.

Let us consider $i = 1, \ldots, N$ agents who order to buy or offer to sell a certain asset. Only a single asset is considered. Its supply, quality, usefulness, etc., are not taken into account. Whether an agent orders to buy or offers to sell depends on his market expectation $m_i(t)$ about the current price. If he thinks the asset is overpriced $m_i(t) = -1$ he offers to sell, if he thinks it is more worth than the current price $m_i(t) = 1$, he buys. To determine what an agent regards as the appropriate price constitutes the crucial part of the model. For simplicity it is assumed that every agent follows the price evolution from