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Evolutionary model of stock markets

Joachim Kaldasch *

EBC Hochschule Berlin, Alexanderplatz 1, 10178 Berlin, Germany

HIGHLIGHTS

- A stock market can be considered as a self-organized dynamic system.
- Then the short-term price has the form of a logistic (Laplace-) distribution.
- The long-term return distribution is a Laplace–Gaussian mixture.
- The mean price expresses the competition between stocks.
- Fisher–Pry plots are a useful tool to study this competition.

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ABSTRACT

The paper presents an evolutionary economic model for the price evolution of stocks. Treating a stock market as a self-organized system governed by a fast purchase process and slow variations of demand and supply the model suggests that the short term price distribution has the form a logistic (Laplace) distribution. The long term return can be described by Laplace–Gaussian mixture distributions. The long term mean price evolution is governed by a Walrus equation, which can be transformed into a replicator equation. This allows quantifying the evolutionary price competition between stocks. The theory suggests that stock prices scaled by the price over all stocks can be used to investigate long-term trends in a Fisher–Pry plot. The price competition that follows from the model is illustrated by examining the empirical long-term price trends of two stocks.

1. Introduction

Financial markets are one of the most intensive economic research fields. Great advantages have been achieved by analyzing time series of the price and return of financial assets. Besides the standard asset price model based on geometric Brownian motion [1,2], stochastic volatility and multifractal models inspired by turbulence [3,4], multi-timescale, scaling and various types of self-similar theories [5–10] have been established. Also multi-agent models [11,12], Auto-Regressive-Conditional-Heteroskedastic (ARCH) and Generalized-ARCH (GARCH) models [13–15] are developed to describe the dynamics of financial markets [16,17].

The aim of this paper is to extend this mainstream research by establishing an evolutionary economic approach for the stock price evolution. The key idea of this model is that financial assets like other commodities are subject to competition in the trading process. This view has been emphasized in particular by Modis suggesting that stocks are in competition for investors' money [18]. He empirically showed that the stock price evolution can be considered to consist of a sequence of logistic growth periods [19]. Following Modis this finding indicates the price competition of stocks.

* Tel.: +49 303929326.
E-mail addresses: jo@kaldasch.de, joachim.kaldasch@international-business-school.de.

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The presented consideration deviates from previous stochastic models by treating a stock market as a dynamic self-organized system in which the dynamics of demanded (desired) and supplied (available) units and the properties of the purchase process determine the price and return distributions of stock shares. The model is based on the idea that the purchase process of stock shares can be treated as a "reaction" between demanded and supplied units while the "reaction velocity" is the purchase rate. The purchase rate is in turn assumed to be governed by the law of mass action, i.e. it is proportional to the product of the number of available and demanded units. These ideas allow establishing price and return distributions of stocks which are in agreement with the main characteristics of empirical data. It also suggests that the long term mean price dynamics of stocks is governed by a Walrus equation [20]. Since this equation can be formulated as a replicator equation it can be shown that the long term price evolution of stocks suffers from competition. The focus of this paper is on a theoretical investigation of this price competition and a comparison with empirical stock price data.

The paper is organized as follows: In the second chapter the dynamic relations of a self-organized stock market are established. A short term price distribution that follows from the price dependent structure of demanded and supplied units and the corresponding purchase rate is derived. In the following chapters it is shown how the Walrus equation for the mean price of a stock can be obtained from perturbations of the distributions of demanded and supplied units and the transformation into a replicator equation. The next two chapters focus on the consequences of this relation. They discuss the price competition between stocks and show that the return distribution can be expected to have the form of Laplace–Gaussian mixture distributions. In order to compare the model with empirical data the price competition of two stocks is examined followed by a conclusion.

2. The model

2.1. Stock market dynamics

The model is established for a closed speculative stock market. Agents, called investors (traders), purchase and sell stock units. In order to describe this market a number of variables have to be introduced. We want to characterize the number of investors by A and indicate them with index i, while the number of stocks is termed B and are indexed by j. The number of units (shares) of the jth stock purchased per unit time by the ith investor is indicated by y_{ij}. The number of units per unit time of the jth stock sold by the ith investor is denoted by y'_{ij}. The total number of purchased units per unit time of the jth stock $\tilde{y}_j$ must be equal to the total number of sold units per unit time $\tilde{y}'_j$ in the trading process[1]:

$$\tilde{y}_j = \sum_i^A y_{ij} = \sum_i^A y'_{ij} = \tilde{y}'_j. \tag{1}$$

The purchase process is regarded as a number of stochastic events where demanded (desired) and supplied (available) shares meet and are exchanged between investors when they agree about the price p. The corresponding number of demanded shares of the jth stock at a given price is denoted by $n_j(p)$ and the number of supplied shares by $z_j(p)$.

We want to treat a stock market as a dynamic system that is governed by the dynamics of the number of demanded and supplied units and the corresponding purchase rate. The dynamics can be given by conservation laws for both variables. The number of demanded units at a given price p of the jth stock is determined by:

$$\frac{\mathrm{d}n_j(p)}{\mathrm{d}t} = D_j(p) - y_j(p). \tag{2}$$

The number of demanded units increases with the demand rate $D_j(p)$ and decreases with the purchase rate $y_j(p)$. The demand rate $D_j(p)$ characterizes the generation rate of desired units per unit time by investors at a given price. It can be regarded to be a function of many variables. Among them is for example the information flow, risk adversion of investors, the expected profitability of invested money etc.

Also the number of available units $z_j(p)$ offered for a price p is governed by a conservation relation:

$$\frac{\mathrm{d}z_j(p)}{\mathrm{d}t} = S_j(p) - y_j(p). \tag{3}$$

The number of available units increases with the supply rate $S_j(p)$ and decreases when units are purchased with the purchase rate $y_j(p)$. The supply rate $S_j(p)$ represents the generation rate of available shares per unit time.

Further introduced are the total numbers of demanded and supplied units by:

$$\tilde{n}_j = \int_0^\infty n_j(p)\mathrm{d}p; \qquad \tilde{z}_j = \int_0^\infty z_j(p)\mathrm{d}p \tag{4}$$

while the total purchase rate of the jth stock given by Eq. (1) can be obtained from:

$$\tilde{y}_j = \int_0^\infty y_j(p)\mathrm{d}p. \tag{5}$$

[1] Variables and functions related to numbers of stock shares are regarded to be scaled by a large number such that they can be treated as real numbers.