Physica A 415 (2014) 479–492

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

From entropy-maximization to equality-maximization: Gauss, Laplace, Pareto, and Subbotin

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HIGHLIGHTS

- We shift from the paradigm of entropy maximization to a model of social-equality maximization.
- This shift implies that the Laplace distribution is the counterpart of the Gauss distribution.
- In the context of wealth and income, the Laplace distribution manifests an optimized balance between equality and variability.
- Also, in the context of wealth and income, the Laplace distribution describes the emergence of a middle class.
- In the context of financial returns, the Laplace distribution manifests an optimized balance between risk and predictability.

ARTICLE INFO

Article history:
Received 30 June 2014
Received in revised form 21 July 2014
Available online 16 August 2014

Keywords:
Entropy-maximization
Equality-maximization
Pietra mean
Subbotin law
Pareto tails
Laplace-distributed returns

ABSTRACT

The entropy-maximization paradigm of statistical physics is well known to generate the omnipresent Gauss law. In this paper we establish an analogous socioeconomic model which maximizes social equality, rather than physical disorder, in the context of the distributions of income and wealth in human societies. We show that – on a logarithmic scale – the Laplace law is the socioeconomic equality-maximizing counterpart of the physical entropy-maximizing Gauss law, and that this law manifests an optimized balance between two opposing forces: (i) the rich and powerful, striving to amass ever more wealth, and thus to increase social inequality; and (ii) the masses, struggling to form more egalitarian societies, and thus to increase social equality. Our results lead from log-Gauss statistics to log-Laplace statistics, yield Paretian power-law tails of income and wealth distributions, and show how the emergence of a middle-class depends on the underlying levels of socioeconomic inequality and variability. Also, in the context of asset-prices with Laplace-distributed returns, our results imply that financial markets generate an optimized balance between risk and predictability.

1. Introduction

One of the fundamental paradigms of statistical physics is that systems, within the boundary of their limiting constraints, will always tend towards a state of maximal disorder, i.e. *maximal entropy* [1]. A quintessential example of this entropy-maximization paradigm is the omnipresent Gauss law, which is described as follows. Consider a complex system whose output is a real-valued random variable. Further consider the system's constraints to yield outputs with a given mean and a

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http://dx.doi.org/10.1016/j.physa.2014.08.011
0378-4371/

given standard deviation. Under these considerations, what is the output that maximizes entropy? The well known answer to this question is the *Gauss law*.

Shifting from physical systems to social systems, can a model analogous to entropy-maximization apply? The goal of this paper is to address this question, and to present an affirmative answer to it. To that end we shall consider the social system to be nothing less than "the economy" itself, and we shall further consider the system's output to be the ultimate manifestation of socioeconomic actions and interactions: *wealth*.

As noted above, physical systems naturally tend to increase their intrinsic disorder, and thus converge to a state of maximal entropy. On the other hand, in social systems great efforts are invested in imposing order — by political structures, by rules, by regulations, etc. Hence, social systems actually do their utmost to reverse the statistical-physics convergence towards a state of maximal entropy. So, what is the social analogue of entropy? What is the measure, if any, that human societies naturally strive to increase? The answer, which is true since the dawn of civilizations, and which has various descriptions, can be neatly summarized by one single word: *equality*.

Throughout the history of mankind, the single most important social driving force was *socioeconomic inequality*. On the one hand, the rich and powerful have always fought to maintain and to further amass their wealth, thus increasing socioeconomic inequality; this phenomenon is commonly referred to as "money goes to money", "the rich get richer", "economic concentration", "Matthew effect", and "preferential attachment" [2–10]. On the other hand, the masses have always struggled to attain a more egalitarian distribution of wealth, thus increasing socioeconomic equality. The clash between these diametrically conflicting interests shaped the course of human history — via the invention of religions, ideologies, and political systems, and via wars and revolutions.

Left unregulated, a physical system will converge to a state of maximal disorder, and the everyday manifestation of this phenomenon is *decay*. Indeed, every physical object around us will eventually decay and fail, and if we desire otherwise, then we have to invest efforts in order to reverse the decay. The maintenance of our houses, of our cars, and even of our very bodies is nothing but our perpetual rebellion against entropy.

Left unregulated, a social system will converge to a state of maximal order, where power is exclusively concentrated in the hands of one supreme entity — be it a geopolitical empire, an economic monopoly, or an absolute monarch. Indeed, with no opposition – moral, judicial, violent, etc. – there is nothing to stop the powerful from attaining ultimate political and economic domination.

Thus, physical and social systems operate antithetically — the former aiming towards maximal disorder and chaos, and the latter aiming towards maximal order and inequality. With no constraints imposed, physical systems will converge to a perfectly symmetric state characterized by a uniform statistical distribution, whereas social systems will converge to a totally asymmetric state characterized by "the winner takes it all" scenario. It is *constraints* that commonly keep systems – physical and social alike – in an optimized balance between the two extremes of perfect symmetry and total asymmetry. The aforementioned Gauss law is a predominant example of the optimized balance attained by physical systems, and in this paper we explore an analogous balance attained by social systems and by financial markets.

In what follows we construct a general optimization framework intertwining together statistical measures of *deviation* and *randomness*. In general, this optimization framework gives rise to the *Subbotin law* [11–13]. In particular, this optimization framework encompasses the entropy-maximization setting that yields the Gauss law, and an equality-maximization setting that yields – on a logarithmic-scale – the *Laplace law* [14]. Consequently, a *log-Laplace law* is obtained, and this law exhibits the key statistical feature of empirically observed income and wealth distributions: *Paretian power-law tails* governing both the poor and the rich [15–19]. Moreover, the log-Laplace law is foundational in the modeling of the distributions of income and wealth [20–28].

Thus, we establish an equality-maximization model that serves as a system-level explanation for the distributions of income and wealth in human societies. Specifically, our model shows how the log-Laplace law emerges as an optimized balance between two opposing forces: the rich pushing towards greater social inequality, versus the masses pushing towards more egalitarian societies. In other words, we demonstrate how shifting from the physical entropy-maximization paradigm to a social equality-maximization model leads from *lognormal statistics* to *Pareto statistics*. Our model further determines the emergence of a *middle-class*, based on the underlying levels of socioeconomic inequality and variability. In addition, applied in the context of asset prices with Laplace-distributed returns, our model asserts that: *financial markets generate an optimized balance between risk and predictability*.

The paper is organized as follows:

- Sections 2 and 3 give concise reviews of the measurement of socioeconomic inequality and statistical variability — via the notions of the Pietra mean, deviation, and randomness.
- Section 4 introduces an optimization framework that intertwines deviation and randomness, and that yields the Subbotin law.
- Sections 5 and 6 apply the optimization results of Section 4 in the context of the returns of financial assets.
- Section 7 applies the optimization results of Section 4 in the context of the distributions of income and wealth.
- Section 8 studies the log-Subbotin law — which is induced by the emergence of the Subbotin law on a logarithmic scale.
- Section 9 addresses the assessment of the risks incurred by financial assets — via the shift from log-Gauss to log-Laplace extreme-event probabilities.
- Section 10 gives a retrospective summary of the path that was traversed along this paper.