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Agent based models for wealth distribution with preference in interaction

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HIGHLIGHTS

- We propose a set of conservative wealth exchange models.
- Three parameters α, β and γ are introduced to mimic real trading.
- Wealth distribution, network properties and activity, etc., have been studied.
- Phase transition and other interesting features are presented.
- Correspondence to real data is shown for different combinations of α, β and γ.

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ABSTRACT

We propose a set of conservative models in which agents exchange wealth with a preference in the choice of interacting agents in different ways. The common feature in all the models is that the temporary values of financial status of agents is a deciding factor for interaction. Other factors which may play important role are past interactions and wealth possessed by individuals. Wealth distribution, network properties and activity are the main quantities which have been studied. Evidence of phase transitions and other interesting features are presented. The results show that certain observations of the real economic system can be reproduced by the models.

1. Introduction

One of the main objectives of several models in econophysics is to reproduce the Pareto tail or power-law tail in the wealth/income distribution in several economies [1]. According to the Pareto law, the probability that the income/wealth of an agent is equal to m is given by,

$$P(m) \sim m^{-(1+\nu)}, \qquad (1)$$

where ν is called the Pareto exponent. The value of the exponent usually varies between 1 and 3 [2–8].

Some of the models proposed to yield the above distribution are inspired by the kinetic theory of gases which derives the average macroscopic behaviour from the microscopic interactions between molecules. Agents can be regarded as molecules and a trading process can be regarded as an interaction between them. In a typical trading a pair of traders exchange wealth,

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respecting local conservation of wealth in any trading [9–14], similar to an elastic collision between molecules. Consequently, the total wealth remains conserved. These agent based models have a microcanonical description and nobody ends up with negative wealth (i.e., debt is not allowed). Thus, for two agents i and j with money $m_i(t)$ and $m_j(t)$ at time t, the general trading process is given by:

$$m_i(t+1) = m_i(t) + \Delta m; \qquad m_j(t+1) = m_j(t) - \Delta m; \tag{2}$$

time t changes by one unit after each trading. The advantage of such models is that here dynamics at an individual level can be studied. In a simple conservative model proposed by Drăgulescu and Yakovenko (DY model hereafter) [10], N agents exchange wealth or money randomly keeping the total wealth M constant. The steady-state ($t \to \infty$) wealth therefore follows a Boltzmann–Gibbs distribution: $P(m) = (1/T)\exp(-m/T)$; $T = M/N$, a result which is robust and independent of the topology of the (undirected) exchange space [12].

An additional concept of *saving propensity* was introduced first by Chakraborti and Chakrabarti [11]. Here, the agents save a fixed fraction λ of their wealth when interacting with another agent. This results in completely different types of wealth distribution curves, very close to Gamma distributions [15–17] which fit well to empirical data for low and middle wealth regime [8]. The model features are basically similar to Angle's work [18]. In a later model proposed by Chatterjee et al. [19] it was assumed that the saving propensity has a distribution, i.e., λ's are now agent dependent and this immediately led to a wealth distribution curve with a Pareto-like tail. Apart from these gas-like models, there are several other models of the wealth distribution. Some of these models depend on the stochastic process [20,21] which cannot be realised as a real trading process. Another model is the Lotka–Volterra model where wealth of an agent at a particular step depends on their wealth in the previous step as well as the average wealth of all agents [22,23]. The main problem in all these models is that here wealth exchange between agents is not allowed and therefore leads to a situation far from reality.

Although wealth distribution is one of the most important features for which the models had been proposed, there are other interesting characteristics of the market which a model should be able to reproduce. Financial institutions are seen to exhibit some interdependence and links are formed among them depending on several economic factors leading to the network structure. In Refs. [24,25] the problem of network formation in a financial system has been addressed. One can then study the network like features, e.g., the kind of clusters which are formed among agents and the behaviour of the degree distribution for better explanation of several economic phenomena. Some real data are available to this respect. It has been shown that within a small interval of time most clusters are of size 2 [26,27] which can be termed as 'dimerisation'. Another observation is regarding the activity, i.e., the distribution of the volume of individual trade that also follows a power law with an exponent $\simeq 4.3$ [28]. These features suggest that one needs to introduce some preference in the interaction between agents.

In almost all the wealth exchange models, the interacting agents are selected randomly and any two agents have equal probability to interact. In this paper, we incorporate preferential attachment to agents for interaction as well as in the choice of agents in some cases. Such preferences need not be limited to geographically nearby neighbours. In Ref. [29], a preference in the selection of agents (according to their wealth) had been considered; however, the interacting agents were uncorrelated otherwise.

To obtain an optimised kinetic exchange model for trade, several features have to be incorporated. Our basic assumption is that two agents will interact only when their wealths are "close". So in the simplest model, only such a feature is incorporated in an otherwise DY like model. More features have been added to obtain results closer to reality. In all the models wealth distribution, network features and other properties are studied.

2. Quantities calculated

We consider kinetic exchange type models where the interactions are of DY type. The simulation is done for a maximum of $N = 1024$ agents. Initially the total money M is distributed among the agents randomly. The stationary state is obtained after a typical relaxation time by checking the stability of the wealth distribution in the successive Monte Carlo (MC) steps, where one MC step is equivalent to N pairwise interactions. The wealth distribution is obtained by averaging over a finite but large number of timesteps. Finally the configurational averaging is done over a number of realisations to obtain the wealth distribution.

Results for the following features have been presented in the paper:

1. Wealth Distribution: $P(m)$ (already introduced in Section 1).
2. Degree Distribution: the number of agents with whom one particular agent interacts within one MC timestep, averaged over all timesteps is the degree of an agent. $D(k)$ denotes the probability that an agent has degree k.
3. Activity Distribution: activity distribution is defined as the number of transactions made by one individual in one MC timestep, averaged over all timesteps. We use $Q(A)$ to denote the activity distribution.
4. Average degree with wealth m: $d(m)$, the average degree of an agent with money m is also calculated to investigate whether the degree is correlated to wealth.

In all the cases, we have taken $M = \sum_{i=1}^{N} m_i$ to be equal to N.