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The hydrodynamic description for the system of self-propelled particles: Ideal Viscek fluid

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HIGHLIGHTS

- We derive Kinetic equations for Vicsek-like system of self-propelled particles.
- We get hydrodynamic equations for the limit of low noise.
- We consider a 1D case for the hydrodynamics of self-propelled fluid.
- The problem of viscosity of such fluid is discussed.

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ABSTRACT

We use the method of the microscopic phase density to get the kinetic equation for the system of self-propelled particles with Vicsek-like alignment rule. The hydrodynamic equations are derived for the ordered phase taking into account the mean-field force only. The equation for the hydrodynamic velocity plays the role of the Euler equation for the self-propelled Vicsek fluid. The hydrodynamics of such ideal self-propelled fluid demonstrates the dynamical transition from disordered initial state to the completely ordered motion. To take the noise into account we consider how the framework of the local equilibrium approximation affects the hydrodynamic equations and the viscous tensor and show that in such approximation the shear viscosity vanishes.

Introduction

The study of collective phenomena in the systems of active agents is an important and fast developing field of statistical physics due to the potential application to the dynamics of living matter [1,2]. This phenomenon is widely observed in nature at different levels of organization including the synchronicity in insects behavior, bird flocking, traffic and complex social behavior [1,3–5]. These systems are essentially non-equilibrium and their dynamics is not strictly Hamiltonian due to the information exchange which includes in general not only the positions of the neighbors but also their velocities.

There are two classes of models that are widely used. Here we adopt the terminology of the works [6,7]. The first class can be called "dynamic". In the models of this class the energy exchange between moving agents and an external source (energy depot) takes place [8] (for the recent review see also Ref. [9]). This class is well suited for the description of self-propelled

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microorganisms immersed in an external fluid, which causes the additional hydrodynamic interaction between agents. The driving forces are mostly due to the gradients of the external factors meaningful for the living organisms such as light, food concentration etc. [10].

The second class is formed by the "spin-like" models of self-propelling particles (SPP) since the speed of particles is constrained to be constant. Thus the velocity plays the role of spin in lattice models of statistical physics. These models are essentially non-holonomic because of the nontrivial control of the angular velocity of a particle rather than its speed. The models of this class are aimed for the description of the cooperative behavior due to the exchange of information between the agents [11]. The most prominent and minimal model of this class was introduced by T. Vicsek and collaborators in Ref. [12]. Due to its simple formulation yet nontrivial character it plays the role analogous to the Ising model in the theory of phase transitions. We call it the Standard Vicsek Model (SVM) [2]. It has generated a series of publications [6,7,13–16] because of its simplicity and rich behavior. Recently, a promising approach which unifies these two classes has been put forward in Ref. [17] by the realization of the Viscek type of interaction via dipole–dipole potential in motile colloids.

There are many questions that still need to be clarified for the model. The Vicsek model can be modified to include nematic alignment [16,18], non-metric interactions [19]. Besides, it can be extended by the inclusion of obstacles as the heterogeneous environment [20,21]. The type of phase transition is one of the main open problems. It is not known exactly either it is continuous [22–25] or discontinuous [15,26–28] and whether this depends on the type of the stochastic perturbation. The mean-field approximations of similar network model [23] and of the Vicsek model itself [29] demonstrate the dependence of the character (continuous or discontinuous) of the transition on the type of noise. Such feature is known for another seminal model of synchronization — the Kuramoto model [30,31]. The SVM can be thought as the dynamic version of the Kuramoto model and at least in the mean-field approximation shares qualitatively the same dependence of the transition (sub- or super-critical) on the type of the noise [32].

Approaching this problem theoretically leads to the formulation of the proper kinetic equation which adequately reflects the basic physical mechanism of self-organization and justifies the hydrodynamic equations proposed previously from phenomenological reasoning [13,33,34]. The first attempt to do this was launched by E. Bertin et al. [35,36] and T. Ihle [37,19]. In Bertin's approach the kinetic equation was obtained following standard Boltzmann derivation using two-particle collision integral. This gave the explicit expressions for the coefficients in the hydrodynamic equations. This approach uses disordered state as the zero-order approximation to construct the distribution function. It shows good agreement with numerical results close to the transition point. Recently this approach was assessed in Ref. [38], pointing at the troubles of the two-collision assumption.

In Refs. [37,19] the Liouville formalism was used as the starting point. The factorization of exact N-particle distribution function to the product of one-particle functions lead to the kinetic equation for the one-particle function. The obtained kinetic equation took into account multi-particle collisions but neglected correlations between particles, which are very important in the SVM. Note that the shear viscosity term appeared in all approaches. But until now no experimental (numerical) evidence of the viscous shear has been reported for the SVM in bounded regions. In Ref. [39] the influence of external shear flow has been studied for the SVM but it has no connection with the inner viscosity of the spp-fluid and can be considered as the specific case of the extrinsic noise [15,40].

In this paper we use well-known method of the microscopic phase density [41]. In the framework of this approach the kinetic equation is constructed directly on the basis of equations of motion without reference to N-particle distribution function. To the best of our knowledge firstly such approach for the self-propelling system with Vicsek-type of interaction was regularly used by P. Degond and S. Motsch in Refs. [42,43]. In these works the hydrodynamic equations for the Vicsek fluid were proposed [44,45]. The method of microscopic phase density, on one hand, is well known from the statistical mechanics and was used to describe, for example, plasma, and on the other hand is different from the Boltzmann approach, or an approach that is based on the factorizing of the N-particle distribution function that are used in Refs. [35–37,19]. Boltzmann approach, and factorization both assume that the system is diluted. The MPDF approach shows a way to take into account more complicated correlations. In this paper we consider only the simplest case, deriving an equation analogous to the Euler equation for the usual fluid. The question of the ideal limit of the Vicsek fluid has never been posed before. We show that such limit is an important starting point for the hydrodynamics of the self-propelled fluid.

We use the basic equation for the microscopic phase density functional as the starting point because of its direct connection with the equation of motion. Our prime interest is to get an equation analogous to the Euler equation, but for the self-propelled fluid from the basic equations of motion, and to compare it with known results: phenomenological as well as derived from kinetic approaches.

The paper is organized as follows. In Section 1 we derive the equation for the microscopic phase density functional of the SVM which leads to the formal kinetic equation with the corresponding collision terms. Then we obtain the hydrodynamical equations. In Section 2 we consider the hydrodynamical limit of the ideal self-propelled Vicsek-like fluid. By the ideal fluid we understand a fluid where the correlations are neglected though the particles interact with each other via the self-consistent field. In Section 3 we consider local equilibrium approximation as the simplest way to take noise into account. We show how it changes the hydrodynamic equations, and compare our coefficients with the ones from Ref. [36]. We discuss the question about the existence of the shear viscosity for such fluid. There we study the viscosity of this fluid from the theoretical point of view using the equations derived in previous sections. We postulate the problem of numerical simulations of Couette flow for Vicsek-like model and discuss preliminary results. The analysis of the results and problems for future studies are given in the concluding section.