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Parameter estimation by fixed point of function of information processing intensity

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HIGHLIGHTS

- We present a new method of estimating the dispersion of a distribution.
- Only part of available data is relevant for the parameters estimation.
- We use function that measures information processing intensity.
- Information processing intensity has a maximum at its fixed point.
- Fixed-point equation is used to estimate the dispersion of a distribution.

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ABSTRACT

We present a new method of estimating the dispersion of a distribution which is based on the surprising property of a function that measures information processing intensity. It turns out that this function has a maximum at its fixed point. Fixed-point equation is used to estimate the parameter of the distribution that is of interest to us. The main result consists in showing that only part of available experimental data is relevant for the parameters estimation process. We illustrate the estimation method by using the example of an exponential distribution.

1. Introduction

In the case of effective collection of statistics, one of the priorities is to obtain information according to the principle of "as much as possible as quickly as possible". We analyse the process of capturing packets of any data in this context. It is assumed that the information measure of these packets is a real random variable having a distribution with a finite dispersion. We are interested in characterising this type of distribution and estimating its dispersion. The method of capturing data packets (and also of making inferences about the distribution of information density for all of the packets based on measurement of these packets) should be optimal, i.e. it should only take into account the necessary part of such packets, i.e. those whose information measure is one-sidedly bounded by a certain constant a. The above assumptions lead to the development of a *function of information processing intensity* which has a surprising property, i.e. it has a maximum at its fixed point. Fixed-point equation is used to estimate the dispersion of a distribution.

Fixed-point theorems constitute a fascinating object of study. Since its beginnings, the development of fixed-point theory has been connected with its numerous applications in other fields of mathematics as well as in game theory and economics [1,2]. Naturally, fixed-point theorems are also used to learn about the distribution of a characteristic within a population based on data obtained from a sample, which is an incredibly important problem in the natural and social

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sciences [1,3]. It is worth emphasising that the estimation method which is presented below can be interpreted in an intuitive, clear and interesting way, by means of maximising the function of information processing intensity, which corresponds to the effectiveness of this process.

2. Information processing intensity

In order to illustrate this problem, let us use the example of an abstract relay **A** which captures packets of data (information). Information packets that have an information value which is too low (or, analogously, too high) are rejected, whereas others are forwarded. This mechanism is analogous to the way in which a high-pass (or low-pass) filter functions. Let us assume that the information measure of information packets that are captured by **A** is a real random variable having a distribution that is characterised by parameter s. Let us also assume that the actions of capturing and rejecting a packet of data as well as the process of forwarding the packet of data that has been captured take the same amount of time.

We denote by I_{in} a random variable which describe the value of information of each single non-filtered package. The probability density function of such a random variable is $\text{pdf}_\gamma(x)$ (γ is the parameter of the distribution). I_{out} is another random variable which measures the contents of the information package of intercepted (filtered) and transferred (emitted) information. We also need to define a random variable (takes values from the set of the natural numbers) T which measures the total time of interception and transfer of the intercepted information package:

$$T = T_{\text{in}} + T_{\text{out}}, \tag{1}$$

where T_{in} is a random variable which represents the time of interception of one information package and T_{out} represents the time of transfer of the same package. We assume that $E(T_{\text{out}}) = 1$, which means that the packages are emitted immediately.

Because of the above assumptions, the measure of information processing intensity is:

$$\rho = \frac{|E(I_{\text{out}} - I_{\text{in}})|}{E(T)}. \tag{2}$$

We assume that $E(I_{\text{in}}) = 0$. This assumption can be interpreted as filter calibration. We denote by a the data cut-off (rejection) parameter (if the information measure of the information package is less than a then such a package is rejected). The number a does not reflect any physical property. Our model is an abstraction of the investigated problem. An analogy can be discerned here to classical acoustic filters (electronic or mechanical). There exists a boundary frequency (amplitude) such that the filter blocks the signal below (or above) it.

We assume that the symbol (*sentence*) is equal to 1 if the sentence is true. In other cases it is equal to 0 (Iverson bracket). We denote by q the probability of rejection and by p the probability of interception of the information package (of course, there is always $(p + q = 1)$):

$$\begin{aligned} q &:= E([x < a]) = \int_{-\infty}^{a} \text{pdf}_\gamma(x+m)\text{d}x, \\ p &:= E([x \geq a]) = \int_{a}^{\infty} \text{pdf}_\gamma(x+m)\text{d}x, \end{aligned} \tag{3}$$

where m is the first moment of random variable I_{in}. In the presented model, it is convenient to shift the domain of function $\text{pdf}_\gamma(x)$ so that the first moment of shift variable distribution is equal to 0. We can calculate the expected time of interception of the information package:

$$E(T_{\text{in}}) = (1-q) + 2q(1-q) + 3q^2(1-q) + \cdots = \frac{1}{1-q}.$$

If we take into account (3) and $E(T_{\text{out}}) = 1$ then it is easy to obtain:

$$E(T) = 1 + E([x \geq a])^{-1}. \tag{4}$$

The random variable $I_{\text{out}} = [I_{\text{in}} \geq a]\, I_{\text{in}}$ has a probability density function $\text{pdf}_\gamma(x+m)$ which is cut off to the field $[a, \infty)$:

$$\frac{[x \geq a]}{E([x \geq a])}\, \text{pdf}_\gamma(x+m). \tag{5}$$

If we put an expected value of random variable I_{out} and (4) to (2) then we get the formula for information processing intensity:

$$\rho(\text{pdf}_\gamma, a)_{right} := \rho(a) = \frac{\int_a^\infty x \cdot \text{pdf}_\gamma(x+m)\text{d}x}{1 + \int_a^\infty \text{pdf}_\gamma(x+m)\text{d}x}. \tag{6}$$