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Reconstruction of network structures from marked point processes using multi-dimensional scaling

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ABSTRACT

We propose a method of estimating network structures only from observed marked point processes using the multi-dimensional scaling. In this method, first, we calculate a spike time metric which quantifies a metric distance between the observed marked point processes. Next, to represent a relationship among point processes in the Euclidean space, we apply the multi-dimensional scaling to the metric distance between point processes. Then we apply the partialization analysis to the obtained coordinate vectors by the multi-dimensional scaling. As a result, we can estimate the network structures from multiple point processes even though the elements have many common spurious inputs from the other elements.

1. Introduction

Nonlinear dynamics could be a possible source for producing complex behavior. Such behavior also depends on connectivity between elements in a dynamical system, or topology between elements. Thus, to analyze, complex behavior produced from nonlinear dynamical systems, it is a key factor to understand both the nonlinear dynamics and their network structures.

Although it is not so easy to investigate the interactions directly, recent developments in measurement technique makes it possible to observe multivariate time series. Then it is possible to estimate the network structure through the multivariate time series.

If an observed time series is continuous and smooth, and sampled by a fixed interval, the network structure can be estimated through statistical measures [1–3] applied to the continuous time series. However, the nonlinear dynamical systems are often observed as an event sequence, and it is difficult to directly apply the conventional statistical measures [1–3] to such an event sequence. These facts indicate that one of the important issues is to develop a method to estimate network structures even if we only observe such event sequences.

To resolve the above-mentioned issue, we have already proposed an estimation method of connectivities between elements only from observed multiple point processes. In the method, we used two measures, the spike time metric coefficient and the partial spike time metric coefficient [4]. In this paper, we propose a more effective estimation method by using the multi-dimensional scaling [5,6].

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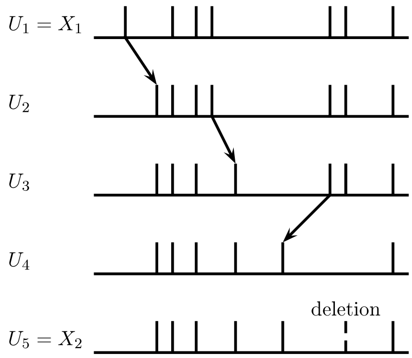

Fig. 1. An example transformation of the spike sequence X_1 into X_2.

First, we proposed a method of estimating network structures from spike sequences as simple point processes. In the proposed method, we measured distance between spike sequences by using the spike time metric that is proposed by Victor and Purpura [7]. Then we applied the multi-dimensional scaling [5,6] to the distance between spike sequences obtained by the spike time metric to represent a relationship among spike sequences in the Euclidean space. By using the multi-dimensional scaling [5,6], we can obtain position vectors of each spike sequences in the Euclidean space. It means that the obtained position vectors can be represented by a linear relationship among the spike sequences. Namely, the linear regression model can be applied to the obtained position vectors. Finally, to estimate connectivity between elements, we used a partial directed coherence which is based on the linear regression model and partialization analysis. To check the validity of the proposed method, we applied the proposed method to observed multiple spike sequences from a mathematical neuron model. In numerical simulations, we show that our method can estimate neural network structures even though the neurons have many common, or spurious, inputs from the other neurons.

In addition, we extended our method to analyze the system that produces marked point processes. It is an important issue to treat observed event sequences as marked point processes, which means that the observed event sequences have event timings and additional information. In the real world, additional information with event sequences can be essential for several complex phenomena, for example, seismic events, transactions in stock markets and so on [8,9]. Such sequences are often referred as the marked point processes. If we can use not only the information of the event timings but also the additional information, we can estimate more precisely the network structures. However, it is not so easy to treat such marked point processes directly. Then, in this paper, we also proposed a method for estimating network structures only from the marked point processes. Different from Ref. [9], using the modified measure and the multi-dimensional scaling, we proposed a new method for estimating network structures from marked point processes. To check the validity of the proposed method, we also conducted numerical experiments and evaluated the performance of the proposed method by using mathematical models: coupled Rössler systems [10].

2. Spike time metric

Spike time metric (STM) [7] is one of the statistics to quantify a distance between two point processes. In the STM, two operations are used to quantify the distance between two point processes. The first operation is deletion or insertion of a single spike. In this operation, the costs are both unity. The second operation is a movement of a single spike. In the second operation, the cost is defined to be proportional to an interval with which the single spike is moved.

If two point processes U and U' are identical except for a single spike that occurs at t_U^i in U and $t_{U'}^j$ in U', the cost of the movement is defined as $c(U, U') = q|t_U^i - t_{U'}^j|$, where q is a positive parameter that determines relative sensitivity of the metric to spike timing [11]. Then, a metric distance between two point processes X_1 and X_2 is defined as follows:

$$D(X_1, X_2) = \min \left\{ \sum_{k=1}^{K-1} c(U_k, U_{k+1}) \right\}, \tag{1}$$

where $X_1 = U_1, X_2 = U_K$ and $U_1, \ldots, U_K$ are elementary steps from X_1 to X_2. The metric distance between two point processes is the minimum total cost of a set of elementary steps that transforms the spike sequence X_1 into X_2. If the distance takes a small value, it means that the spike sequence X_1 is similar to X_2. Fig. 1 shows an example of transforming spike sequence X_1 to X_2.

Here, it is important to decide the parameter q to apply the STM to point processes. If the parameter $q = 0$, the distance simply measures the difference in the number of spikes, because the cost of movement is zero. If the parameter $q > 0$, the parameter q determines which operations, deletion and insertion or the movement, have priority. Let us consider two spike sequences X_a and X_b, each of which consists of only a single spike that occurs at t^a and t^b, respectively. Then, two paths are possible, deletion and insertion of a spike whose cost is two, or the movement of a single spike whose cost is $q|t^a - t^b|$. If