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Systems of companies with assets in common: Determining true interests

Salvador Cruz Rambaud [a,*], Derek J.S. Robinson [b]

[a] Departamento de Economía y Empresa. Universidad de Almería, 04071 Almería, Spain

[b] Department of Mathematics. University of Illinois at Urbana-Champaign, IL 61801, USA

HIGHLIGHTS

- We model the digraphs of affiliations of companies with only purely algebraic tools.
- We determine the true interests in complex business combinations with an algorithm.
- We generalize the "Theorem of Influence" of Crama, Defourny and Gazon.
- We solve complicated situations involving large numbers of companies with overlapping interests.
- Potential use of our algorithm with available current software.

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ABSTRACT

We consider systems of companies which have financial interests in each other and develop principles by which the real interest of one company in another can be determined. The paper develops an algorithm to calculate true interests from a weighted digraph that encodes the direct and indirect mutual holdings of the companies.

1. Introduction

In complex systems of financial entities it is important to be able to determine the true interest of one entity in another one. This capacity is essential for the government (in particular in regard to taxation matters) and banks, among others institutions. In this way one can regard the true interest as the effective level of influence that the investor can exert on the investee, that is, the real proportion of the investee's equity that is owned by the investor. Usually the true interest must be determined in situations of multiple ownership levels which involve direct and indirect (reciprocal or mutual) interests [1,2].

Usually, the approach for direct and indirect ownership computation starts from the classical input–output matrix. Nevertheless, Defourny and Thorbecke [3] introduce the Social Accounting Matrix (SAM) "as a general equilibrium data system linking, among other accounts, production activities, factors of production and institutions (companies and households)". These authors determine the direct influence of firm i on another company j transmitted through an arc or an elementary path. Later they derive the indirect effects induced by the circuits adjacent to the path (defined as those circuits which have one or more poles in common with the path). This is the so-called total influence, and then the "global influence from pole i to pole j simply measures the total effects on income or output of pole j consequent to an injection of one unit of output

* Corresponding author. Tel.: +34 950015184; fax: +34 950015178.
E-mail addresses: scruz@ual.es (S.C. Rambaud), dsrobins@illinois.edu (D.J.S. Robinson).

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or income in pole i''. This concept was provided by Lantner [4] through the well-known ''Theorem of Influence''. Thus, the calculation is based on the multiplicative decomposition of the corresponding multipliers (see Ref. [5]). Our methodology is similar to that of Defourny and Thorbecke. The main difference is that these scholars use calculations with determinants, while our approach is based on the successive application of our formulae which generalize the expressions (16) and (17) in Defourny and Thorbecke. Moreover, our step-by-step procedure can easily be followed through the digraph representing the dependence among firms.

In order to determine the level of ownership and so the shareholders' control, Chapelle and Szafarz [6] present a model for measuring the so-called integrated ownership for pyramids, cross-ownership, rings and other complexes of interrelated firms.

The aim of this article is to define and provide a method for calculating the true interest of one company in another in the context of a complex system of companies with mutual interests. Until now the case of mutual stock holdings has been solved using the following methods:

1. Geometric series [7].
2. Systems of simultaneous equations involving the interests of noncontrolling shareholders [8].
3. Matrix calculus [9].
4. Finite Markov chains [10].

In this article we present an algorithmic solution to the problem of calculating the true interests in complex cross-holdings situations. The algorithm can be applied to a weighted digraph which encodes the mutual dependences among the companies. In very large systems it can be applied to the corresponding matrix using machine computation. A feature of our approach is that it takes into account the presence of cycles, which tend to reduce the true interests of companies.

We remark that the mathematical methods employed here are quite elementary in nature, involving basic digraph theory and simple algebra. The present authors were part of the project (see Refs. [11,12]) that sought to apply algebraic methods to analyze the accounting structure of a single company. The current investigation, on the other hand, seeks to apply algebraic techniques to systems of companies. Once again the power and precision of algebraic concepts and language are shown to be highly effective tools.

The organization of this article is as follows. Section 2 provides the notation and the nomenclature to be used throughout the paper. In Section 3 we establish the basis for the definition of true interest and begin to address the problem of cycles. Section 4 introduces the critical concept of a segment in a path, which is used to deal with the situation when cycles are present. Here a segment of a path is a subpath all of whose edges meet the same set of cycles. In Section 5, after enumerating the principles for computing true interests, several examples are presented in which true interests are computed. Finally, Section 6 summarizes our methods and makes it clear just what assumptions it is based on.

2. Notation, nomenclature and preliminaries

Before introducing the notation, we recall the concept of interest, which already appears in the context of mergers and acquisitions [13,14]. When a company, called the *parent*, controls a part of another company, called the *subsidiary*, the corresponding subsidiary's individual net assets are reported by the parent in an ''equity investment'' account, which represents a percentage of the subsidiary's equity [15]. Indeed this account represents the interest of the parent in the subsidiary.

We will consider a *system of financial entities*, for convenience referred to as companies,

$$\mathcal{S} = \{C_1, C_2, \ldots, C_n\},$$

some of which have interests in other companies in the system. Suppose that company C_i owns directly a proportion p_{ij} of the net equity of company C_j where $0 \leq p_{ij} \leq 1$. It is convenient to define p_{ii} to be 0, so that we can form an $n \times n$-matrix with entries in the interval [0, 1]

$$P = [p_{ij}].$$

Since the sum of the proportions of company C_j owned by other companies cannot exceed 1, the entries of matrix P must satisfy the conditions

$$\sum_{i=1}^{n} p_{ij} \leq 1, \quad j = 1, 2, \ldots, n. \tag{1}$$

These considerations motivate the following definition.

Definition 1. Let $\mathcal{S}$ be a system of companies. A *system matrix* for $\mathcal{S}$ is matrix $P = [p_{ij}]$ satisfying the following conditions:

(i) $p_{ii} = 0$, for every $i = 1, 2, \ldots, n$.
(ii) $0 \leq p_{ij} \leq 1$, for every $i, j = 1, 2, \ldots, n$.
(iii) $\sum_{i=1}^{n} p_{ij} \leq 1$, for every $j = 1, 2, \ldots, n$.

Thus, the relationships among the companies in $\mathcal{S}$ is completely described by the system matrix P. Conversely, such a matrix P determines a system of companies with financial interests in each other as specified by the entries of the matrix.