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Transmission of information at criticality

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HIGHLIGHTS

- Information transmission reaches a maximum when the two cooperative systems are operating near a phase transition.
- An isolated system near a critical point displays ergodicity breaking over extended observation times.
- Ergodicity breaking is temporary.
- The aging time necessary to recover ergodicity depends on the finite size of the system.
- When the two complex systems are at criticality, information transfer happens through intermittent events instead of waves.

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ABSTRACT

We study the problem of information transmission in complex cooperative systems to prove that adaptivity rather than diffusion is the main source of information transport at criticality. We adopt two different cooperative models, the two-dimensional Decision Making Model (DMM), and the one-dimensional Flock Model (FM) inspired by the cooperation between birds. The criticality-induced consensus is intermittently broken by the occurrence of moments of high susceptibility, which we call *free-will states*. We construct a network A based on the DMM and FM that is perturbed by a similar network B. Some units of *A*, called lookout birds, follow the directions of the mean field generated by *B*, while the rest are blind to *B*. When both networks are at criticality a small percentage of lookout birds establish the synchronization between *B* and *A* as a result of the nonergodic nature of the free-will state dynamics.

1. Introduction

The attractive concept of collective cognition [1] is closely connected to the problem of information transfer between the individuals of an organized society while information transport is conjectured to become maximally efficient at criticality [2]. Recently much attention has been given to the role of criticality [3] in a wide range of complex systems, from bird flocks [4] to social groups [5] to neural networks [6] and the brain [7]. At criticality, the short-distance links of Ising-like cooperative models are converted into long-distance interactions, thereby going beyond *locality*, while turning a set of *N* virtually independent units into an organized system behaving as a single individual with extended cognition [8,9].

The work of Ref. [10] shows that criticality generates weak ergodicity breaking [11] and renewal aging [12], two concepts that are gaining popularity from single-molecule tracking experiments inside living cells [13,14]. Weak ergodicity

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breaking and renewal aging, concepts frequently ignored in conventional statistical physics, are currently challenging theoreticians [15] to establish a new perspective of nonequilibrium statistical physics that may account for these important properties frequently emerging in complex systems (see also Refs. [16–18]).

This article is devoted to illustrate how criticality-induced ergodicity breaking and locality violation lead to a form of information transport not requiring the information waves frequently taken for granted (see for instance Refs. [1,4]). Although in the long-time regime ergodicity and stationarity are recovered, this article shows that the free-will states generate synchronization, operating on a much shorter time scale than equilibrium. The time distance between two consecutive free-will states is described by a *brand new* nonstationary survival probability. The brand new distribution tells how long we have to wait for a new event given that we start measuring the time at the instance of an event occurrence. In the absence of an exponential long-time truncation, the brand new distribution would have the scale-free structure

$$\lim_{\tau\to\infty} \psi(\tau) = \frac{\text{const}}{\tau^{\mu}}, \tag{1}$$

with $\mu < 2$, thereby yielding $\langle\tau\rangle = \infty$ and breaking ergodicity. The renewal aging is an important property of this ideal scale-free case [15].

Real systems, including those discussed in this article, depart from this ideal condition, and the scale-free distributions are usually truncated. In the case of this article the exponential truncation is caused by the finite size of the systems under study. Therefore, the theory illustrated in this article refers to the case when the inverse power law distribution density of Eq. (1) is truncated at time T_d. As a consequence, through an extended renewal aging process, the complex systems tend toward the stationary equilibrium correlation function, with a correlation time T_c that is very close to the truncation time T_d. The resilience of these systems to environmental influence is determined by this extended time region of regression to equilibrium, thereby making the role of temporary aging as fundamental as the perennial aging of Ref. [15]. As we shall see hereby, and as discussed in detail in Section 5.2, the long-time properties of synchronization are determined by the nonstationary brand new distribution density and consequently are the result of a theoretical approach going beyond the conventional stationarity assumptions.

The theoretical discussion of this article focuses on the cooperative model of Ref. [10], which leads with analytical arguments to $\mu = 1.5$. In the absence of truncation the waiting time distribution density $\psi(\tau)$ of Eq. (1) becomes slower and slower with age. We invite the readers to consult the recent paper of Ref. [19] to properly appreciate the fact that delaying the beginning of an observation from the preparation time $t = 0$ to the time $t_a > 0$ has the effect of making the decay of $\psi(\tau)$ slower and slower upon increase of t_a. As a result of truncation at time T_d, as we shall see in Section 5.1, the mean time of $\psi(\tau)$ and of the corresponding infinitely aged probability density becomes finite. We shall denote the mean time of $\psi(\tau)$ as T_{new} since it is a property of the brand new nonstationary probability distribution, and we will show there is a separation of time scales in that $T_{new} \ll T_d$. The effect of truncation is to turn the process of perennial aging into an aging process with finite time duration. We restate that it is remarkable that the synchronization between the driven complex network and the driving complex system, an important result of this paper, is realized during this extended process of transition to equilibrium.

2. Cooperative models

We shall shed light into the criticality-induced properties mentioned in Section 1, locality breakdown and the nonequilibrium origin of synchronization, by using two different cooperative models. The Decision Making Model (DMM) of Refs. [10,20,21] and the Flock Model (FM) of Vicsek et al. [22] are both characterized by the occurrence of free-will states, and have been used in an earlier publication to prove that criticality maximizes the efficiency of information transmission [8]. In both models the interaction between units is local, thereby naturally suggesting the existence of information waves that this article proves to be inessential.

2.1. Decision making model

In the DMM, a single unit i has to make a choice between *yes* (State 1) and *no* (State 2), under the influence of its nearest neighbors, according to the transition rates

$$g^{(i)}_{1\to 2} = g \exp\left[-K\left(N^{(i)}_1 - N^{(i)}_2\right)/N^{(i)}\right] \tag{2}$$

and

$$g^{(i)}_{2\to 1} = g \exp\left[K\left(N^{(i)}_1 - N^{(i)}_2\right)/N^{(i)}\right]. \tag{3}$$

$N^{(i)}_1$ and $N^{(i)}_2$ are the number of neighbors in State 1 and State 2 respectively, and the total number of neighbors is $N^{(i)}$. The parameter g corresponds to the rate of information exchange between a unit and its nearest neighbors. The cooperation strength is determined by K. Cooperation here means that if a unit has more neighbors saying *yes*, the transition rates are biased so the unit is more likely to say *yes* as well, and similarly for *no*.

Throughout this paper $g = 0.01$ and the total number of units is set to $N = 50 \times 50$. The units of the DMM are the nodes of a regular two-dimensional (2D) lattice of size $L_D = 50$ with periodic boundary conditions, making $N^{(i)} = 4$ for every unit.