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Nonlinearity effects on conductance statistics in one dimensional disordered systems

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HIGHLIGHTS

- The non-linearity (NL) can either localize or delocalize the electronic states depending on its sign.
- The power-law exponent γ is found to be sensitive to the kind of potential.
- We found a characteristic length Lc separating exponential decaying to power-law decaying eigenstates.
- The conductance probability distribution shows deviation from its log-normal form when the nonlinearity is increased.
- The conductance fluctuations decrease with increasing NL interaction indicating the delocalization of the eigenstates.

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ABSTRACT

We investigate numerically the effect of non-linear interaction on conductance statistics in one dimensional disordered systems with δ peak potentials. It is shown that the non-linearity can either localize or delocalize the electronic states depending on its sign. For an attractive nonlinear interaction, we found that the mean conductance decays as $g \approx L^{-\gamma}$. The exponent γ is found to be sensitive to the kind of the potential. It seems to be independent of the strength of the non-linearity in the case of disordered barrier potentials, while it varies with this strength for well and mixed potentials. The conductance probability distribution shows a deviation from its log-normal form (linear case) when the nonlinearity is increased and the fluctuations of conductance decrease indicating the delocalization of the eigenstates.

1. Introduction

The phenomena of Anderson localization of electrons and the disorder induced metal–insulator transition (MIT) have been studied extensively for decades [1–3] and continue to attract much attention. Scaling arguments [4] predict that almost all eigenstates in one-dimensional (1D) linear uncorrelated disordered systems are localized for any amount of disorder. The electronic transmission coefficient was shown to have an exponential decay [2,3] leading to the absence of transport in an infinite system. The origin of Anderson localization is the interference between multiple scattered electronic waves.

However, recently some models of disorder introducing the correlation [5–7] and the nonlinearity [8–10] have been found to exhibit a number of extended states at particular energies. These discrete extended states have been observed recently in the experiments with GaAs–$Al_xGa_{1-x}As$ random super-lattices [11] and in nonlinear disordered media [12].

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The effect of nonlinear interaction on localization caused by disorder is a subject of both theoretical and experimental interest [13–17]. From the theoretical point of view, we expect new effects to arise due to the competition between the well known localizing effects of the disorder and the delocalizing effect due to the non-linear interaction in an appropriate regime.

Nonlinear interactions are often present in a real system. The nonlinear Schrödinger equation has been studied extensively in recent years and served as a prototype for studying nonlinear phenomena. The origin of the nonlinearity in the Schrödinger equation corresponds to different physical phenomena. In electronic systems, it would correspond to Coulomb interaction while in a superfluid it corresponds to the Gross–Pitaevskii equation. It has been demonstrated recently that above a certain critical strength of nonlinearity, the Anderson localization is destroyed [18]. It was also shown that the competition between nonlinearity and disorder may contribute for some initial conditions to energy localization or for others to energy propagation [19].

In a previous paper [20], we found that the nonlinearity can either localize or delocalize the electronic states in 1D disordered systems depending on the strength and the sign of the nonlinear potential. In periodic systems, the attractive nonlinearity was found to suppress the gap [20]. Physically, a repulsive nonlinear (NL) potential represents the electron–electron interaction, while an attractive one corresponds to the electron–phonon interaction. These interactions are important in various systems such as quantum dots, and super-lattices [21]. It is also found that the nonlinearity can suppress the Wannier–Stark effect caused by the electric field in case of ordered potentials and can give rise to a transition from super-localized to weakly localized states in disordered systems [22]. In a recent paper, we have studied the effect of NL interaction on resonant states of the random dimer model [23]. We found that the NL interaction increases or decreases the resonance bandwidth depending on both its sign and strength.

Intensive works have been devoted to understand the transport properties in nonlinear disordered systems [8–10,13–19] but to the best of our knowledge, the effect of nonlinear interaction on conductance fluctuations and distribution has not been examined previously. The purpose of this work is to investigate the influence of nonlinear interaction on conductance statistics in 1D disordered systems. We calculate the disorder-averaged conductance and its probability distribution for different types of potential. In a previous work [20], we found that the transmission exhibits power-law decay around the band edges of the corresponding periodic system, while for other energies the transmission decays at least exponentially if not faster. It is important to study the behavior of the exponent of the power-law and its dependence on both the nonlinearity strength and the kind of disorder.

2. Model description

The model considered here consists of a single electron with energy E moving along a one-dimensional periodic lattice of $N\delta$-peak potentials equally spaced with a lattice parameter a plus a nonlinear interaction of strength α on each site. The model is described by a time independent discrete nonlinear Schrödinger equation as [24]:

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\sum_{n=1}^{N}(\beta_n+\alpha|\Psi(x)|^2)\delta(x-na)\right]\Psi(x)=E\Psi(x) \tag{1}$$

here $\Psi(x)$ is the single-particle wave function at site x, β_n the potential strength of the nth site. The nonlinear interaction is of δ-function type and with constant strength α. This means that the electron propagates freely between lattice sites and interacts only at $x = n$. The electron energy E is expressed here in units of $\hbar^2/2m$ with m being the electronic effective mass. For simplicity the lattice spacing is taken to be unity throughout this work. The potential strength β_n is a variable picked up from a random distribution with $-W/2 \le \beta_n \le W/2$ for the mixed-potential case, $0 < \beta_n \le W$ for the potential barrier case and $-W/2 \le \beta_n < 0$ for the potential well case (W being the degree of disorder). All the nonlinear physics is contained in the coefficient α. The positive α corresponds to repulsive (electron–electron) interaction while negative α models attractive (electron–phonon or ion) interactions. The lattice parameter is taken for commodity to be unity ($L = N$). The two ends of the chain are assumed to be connected to perfect leads where the wave-functions are plane waves. The wave functions between lattice sites are of the form $\Psi_n = A_n\,\mathrm{e}^{ikx} + B_n\,\mathrm{e}^{-ikx}$ where the momentum is given by $k = \sqrt{E}$. From the computational point of view it is more useful to consider the discrete version of this equation, which is called the generalized Poincaré map and can be derived without any approximation from Eq. (1). It reads [24]:

$$\Psi_{n+1}+\Psi_{n-1}=\left[2\cos\sqrt{E}+\frac{\sin\sqrt{E}}{\sqrt{E}}(\beta_n+\alpha|\Psi_n|^2)\right]\Psi_n. \tag{2}$$

The solution of Eq. (2) is carried out iteratively by taking the two initial wave functions at sites 1 and 2: $\Psi_1 = \mathrm{e}^{-ik}$ and $\Psi_2 = \mathrm{e}^{-2ik}$.

The transmission problem considers an incident wave at $x = L$ with $\Psi(x) = r_0\,\mathrm{e}^{-ik(x-L)} + r_1\,\mathrm{e}^{ik(x-L)}$ for $x \ge L$ and a transmitted wave $\Psi(x) = te^{-ikx}$ for $x \le L$. In order to have a uniquely defined problem the output is fixed $|t^2| = 1$ and then