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A speed feedback control strategy for car-following model

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HIGHLIGHTS

- A speed feedback control mechanism was introduced into car-following system.
- The stability of traffic flow system under different feedback coefficients was discussed.
- The unit step response and phase margins were analyzed based on control theory.

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ABSTRACT

A speed feedback control mechanism was introduced into the system to improve the dynamical performance of the traffic flow. The modern control theory was used to analyze the stability of the system. It is found that the stability region varies with the feedback coefficient proportionally. In addition, the unit step responses in time domain and phase–frequency curves in frequency domain were given with different feedback coefficients in step response diagram and Bode diagram respectively. The overshoot and phase margins are inversely proportional to the speed feedback coefficients in an underdamped condition. The simulations were conducted to verify the validity of the improvement. The conclusion can be drawn that the analytical result and the simulation result are in good agreement with each other.

1. Introduction

Some traffic flow models have been developed to describe the dynamical characteristics of the moving vehicles in the past six decades [1–32]. Car-following model as one of microscopic ones has superiority in describing the motion of every vehicle. Since it was proposed firstly by Pipes in 1953 [1] it was not improved for more 40 years until Bando et al. [2] proposed an optimal velocity model in 1995. After that time, many physicists and scholars were attracted to study car-following model with different theory. Lenz et al. [3] extended Bando's optimal velocity model with an anticipated term and Nagatani et al. [4], Sawada [5] took the next-nearest-neighbor interaction into account in the Bando model. These improved models were more realistically in describing the traffic jam in an unstable region. Jiang et al. [6] and Xue [7] discussed an extended car-following model with a consideration of the velocity difference between the current vehicle and its immediate front one. They found that the velocity difference plays an important role in the phase transition and the traffic congestion. Furthermore, Konishi et al. [8,9], Hasebe et al. [10,11], Ge et al. [12], Li et al. [13], Zhao et al. [14], Tang et al. [15–22], Zhu et al. [23–26], and Peng et al. [29,30] creatively improved and analyzed car-following model with their own viewpoint. Due to these scholars' efforts traffic flow theory was promoted to a high level. Among these outstanding achievements Konishi

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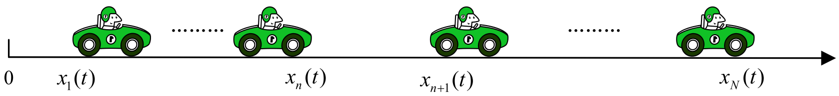

Fig. 1. Illustration of the car-following system.

et al. [8,9] and Zhao et al. [14] mainly focused on analyzing the traffic flow system with a modern control theory. Based on their ideas, Zhang et al. [31] introduced a PD (Proportional–Differential) term into traffic flow model and some new results were obtained through analyzing car-following model by the use of the classical control theory. The performance of the improved model was generally better than those of the previous models but a zero point was generated in the closed loop transfer function because of the PD term. The stability of the traffic flow system was weakened due to the influence of the zero point. In order to eliminate the influence of the zero point a speed feedback control term was used to take the place of PD term in this paper. In the following part we will devote to showing the superiority of the speed feedback control to the PD control for car-following model.

The remainders of this paper are organized as follows. In Section 2 the model was improved and carefully investigated by the use of the modern control theory. In Section 3, the improved model was analyzed with the time domain and frequency domain methods. In Section 4, numerical simulations were carried out to verify the validity of the new model. In Section 5 the summary is given.

2. Model

Assume that all vehicles move one after one on a single lane road without overtaking under periodical boundary condition in an N-vehicle system (in Fig. 1). The road length is L m and $x_n(t)$ denotes the position of the nth vehicle at time t and $n = 1, 2, \ldots, N$, N is the total number of the vehicles in the system. The headways of the nth and Nth vehicle at time t are $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ and $\Delta x_N(t) = L + x_1(t) - x_N(t)$ respectively. Based on Bando's optimal velocity model the motion equation of the system is given,

$$\ddot{x}_n(t) = a \times [V(\Delta x_n(t)) - \dot{x}_n(t)] \tag{1}$$

where a is the sensitivity of the driver generally $a = 0.85\ \mathrm{s}^{-1}$, $\ddot{x}_n(t)$, $\dot{x}_n(t)$ denote the acceleration and velocity of the nth vehicle at time t, and $V(\Delta x_n(t))$ is the optimal velocity function (OVF) of the nth vehicle given as that in Ref. [32],

$$V(\Delta x_n(t)) = \frac{v_{\max}}{2} \times (\tanh(c_1 \times (\Delta x_n(t) - s_c) - c_2) + \tanh(c_1 \times s_c + c_2)) \tag{2}$$

where $v_{\max} = 15.82$ m/s, $c_1 = 0.13$, $c_2 = 1.57$, $s_c = 5$ m which denotes the length of the vehicle.

In order to analyze the stability of the system we define the following steady state variables in the N-vehicle system in Eq. (3)

$$[v_n^*(t), \Delta x_n^*(t)]^T = [v_0, V^{-1}(v_0)]^T \tag{3}$$

where $v_n^*(t)$ and $\Delta x_n^*(t)$ represent the steady state velocity variable and steady state headway variable of the nth vehicle at time t. $v_0 = v_n(0) = V(L/N)$ denotes the homogeneous velocity of the system, $V^{-1}(v_0)$ denotes the homogeneous headway of the system.

Based on modern control theory, if we add small perturbation into the homogeneous traffic flow system, it can be linearized around steady state as,

$$\begin{cases} \dot{X}_n(t) = AX_n(t) + BU_n(t) \\ Y_n(t) = CX_n(t) + DU_n(t), \end{cases} \tag{4}$$

where $X_n(t)$ is system state variable vector. $U_n(t)$ is the input variable vector and $Y_n(t)$ is the output variable vector. These three variable vectors can fully describe the dynamics of traffic flow system.

$$\dot{X}_n(t) = \begin{bmatrix} \dfrac{\mathrm{d}(\tilde{v}_n(t))}{\mathrm{d}t} \\ \dfrac{\mathrm{d}(\Delta\tilde{x}_n(t))}{\mathrm{d}t} \end{bmatrix}, \qquad X_n(t) = \begin{bmatrix} \tilde{v}_n(t) \\ \Delta\tilde{x}_n(t) \end{bmatrix}, \qquad U_n(t) = \tilde{v}_{n+1}(t) \tag{5}$$

where $Y_n(t) = \tilde{v}_n(t)$, $\tilde{v}_n(t) = \tilde{v}_n(t) - v_0$, $\tilde{v}_{n+1}(t) = v_{n+1}(t) - v_0$, $\Delta\tilde{x}_n(t) = \Delta x_n(t) - V^{-1}(v_0)$. A, B and C are the coefficient matrices depending on car-following model. $x_n(t)$ and $\Delta x_n(t)$ are the same as the aforementioned and $v_n(t)$ is the velocity of the nth vehicle at time t.

With linear system theory, we obtain one form of state space matrices from the coefficients in Eq. (1) which can be rewritten as,

$$A = \begin{bmatrix} -a & a\Omega \\ -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D = 0. \tag{6}$$